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## INFEM: An Inference-Finite Element Program For Flow Problems

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INFEM: An Inference-Finite Element Program  
for Flow Problems

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## 1. Introduction

In the present work an inference model producing the input data to the main analytical scheme on the basis of field information is demonstrated. The main body of data concerning the physical parameters are taken from the field of rock mechanics.

Krumbein (3), describes the fundamental techniques used in defining linear inference models. Such techniques are for example the method of least squares, fitting a polynomial of two variables, etc. However, all these techniques fail to provide an evaluation of how good the estimation is. They also exhibit operational difficulties for the nonstationary case. A different inference scheme is proposed by Matheron (5). It is based on a model originally suggested by Krige (4), and is particularly well suited to treat nonstationary cases. A similar moving average technique is presented hereafter, that generates the spatial

distribution of the physical properties of the rock media, defined in the previous chapter. Also the interfacing of the Inference model with the analytical Finite Element model is shown below.

## 2.2 Justification of the Moving Average Technique

Drill hole samples most often produce extreme values which are erratic in their spatial distribution. Figure 2.1 gives an illustration of the variability of some commonly investigated physical parameters. Therefore there is a need for models generating smooth spatial distributions of these parameters. Two such models most commonly used are based on trend surface estimates and moving average estimates.

It is generally observed that moving average estimates are superior to trend surface estimates. In the former case the estimation is exclusively based on adjacent informational sets, the more distant sets exerting no influence at all. In the case of the trend surface estimates, the estimation is based on all known data. Moving average estimates tend to be more stable than corresponding polynomial trend estimates obtained from the same number of data points, especially if the sample points are sparse, Whitten (7). Davis (2), observed that moving average schemes in two dimensions have not been significantly tested yet. Indeed the time series analysis from which this method is derived is not as well developed at the present time, as the regression analysis, for example, on which the trend-surface method is based.

Among the moving average methods, the Kriging (named by French geomathematicians in honor of Krige) seems to be the most advantageous. More specifically the Kriging technique makes optimal use of the given data and provides a measure of the variance of the estimation made at

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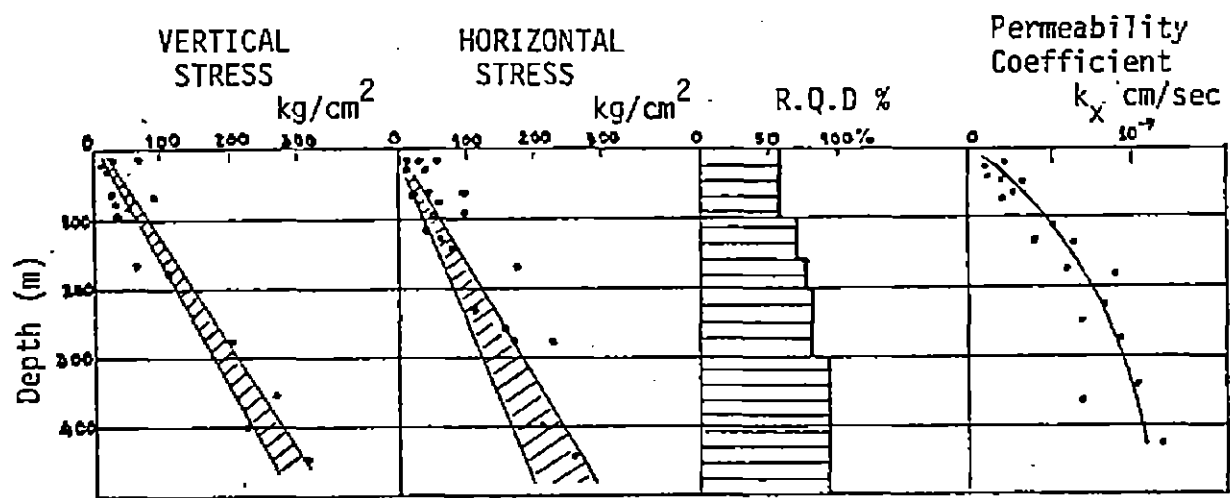


FIGURE 1.1 VARIABILITY OF PHYSICAL PARAMETERS

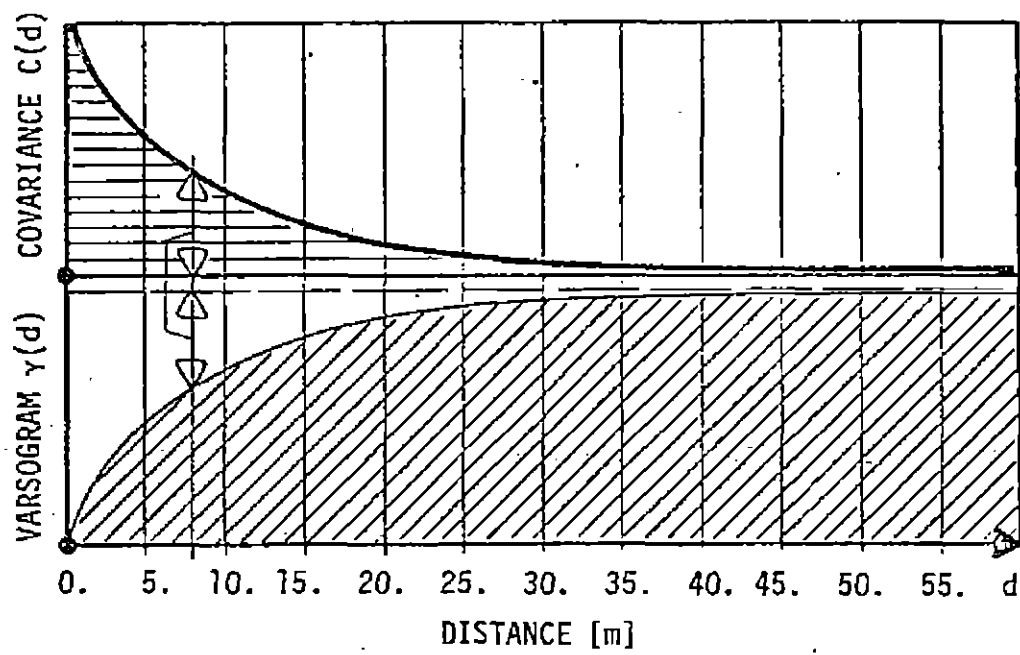


FIGURE 2.2 VARIOGRAM AND COVARIANCE VS. DISTANCE

any specific location. Both these features are essential if the results from the Inference model are to provide the input for an uncertainty analysis. However it is believed that a general method incorporating parts of both techniques may give superior results.

### 3 Implementation of the Inference Model in a Two-Dimensional Geometric Space

#### 3.1 Statistical Model and Corresponding Assumptions

To determine the values of a rock property  $Z(x,y)$  a number of measurements are made on rock samples from bore holes. The set of points where observations are made is indexed by  $\beta$  and  $Z_\beta$  represents the measured value of the random rock property at point  $\beta$ .  $\hat{Z}(x_0, y_0)$  is the estimation of property  $Z$  at a particular point  $(x_0, y_0)$  in the media, evaluated from the measured  $Z_\beta$  values.

One simple way to obtain this estimation is to define  $\hat{Z}$  in terms of the known values  $Z_\beta$  according to a linear combination as follows:

$$\hat{Z}(x_0, y_0) = \sum_{\beta=1}^n b_\beta \cdot Z_\beta \quad (3.1)$$

where:  $Z_\beta$  are the known data points,

$b_\beta$  are unknown weight coefficients to be determined by the Inference model.

An alternate approach would be to estimate the mean value of  $Z(x,y)$ , namely  $\bar{Z}(x,y)$  at point  $(x_0, y_0)$ , according to a linear combination:

$$\hat{\bar{Z}}(x_0, y_0) = \sum_{\beta=1}^n a_\beta \bar{Z}_\beta \quad (3.2)$$

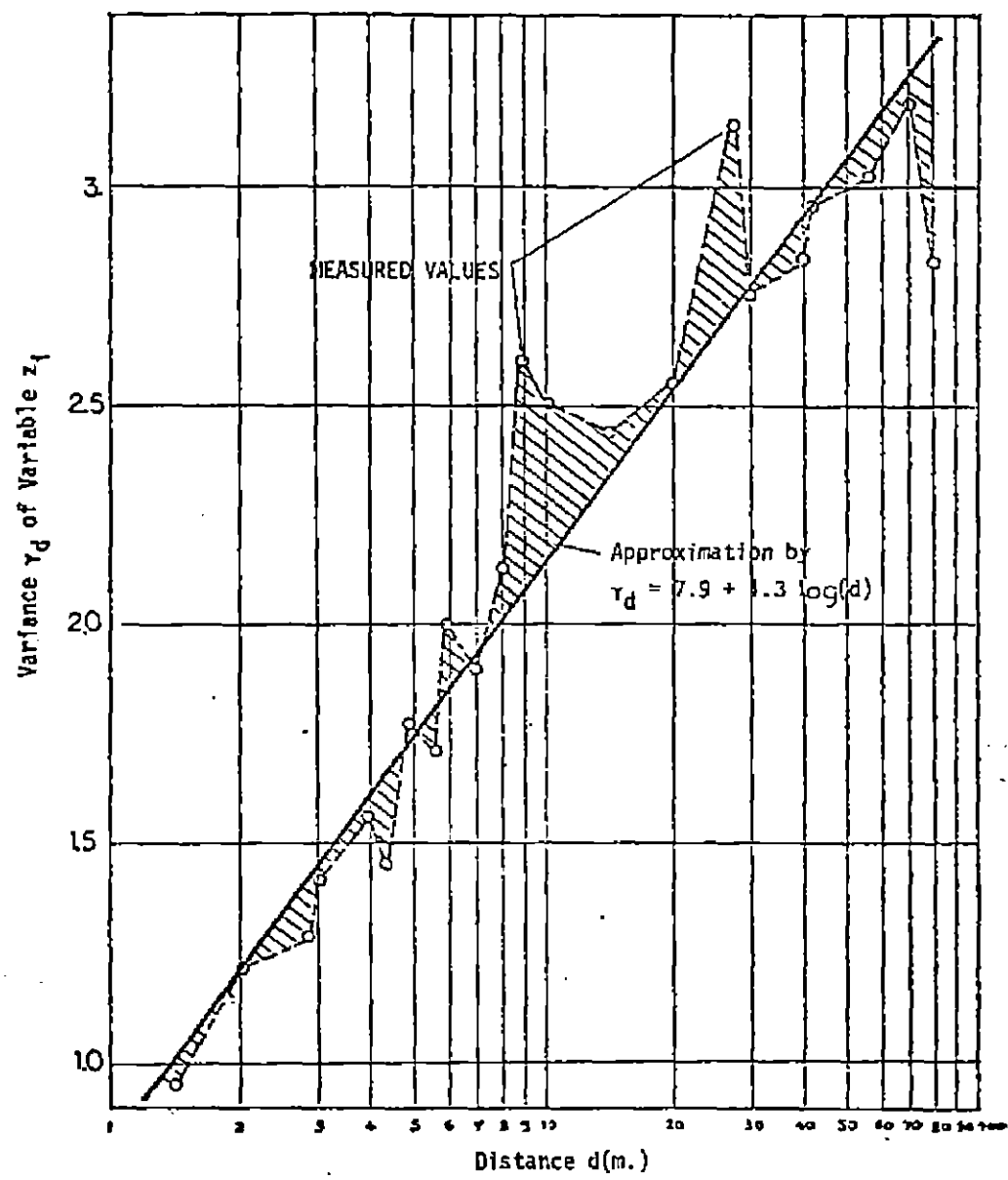


FIGURE 4 APPROXIMATE VARIOGRAM FUNCTION FROM ACTUAL DATA

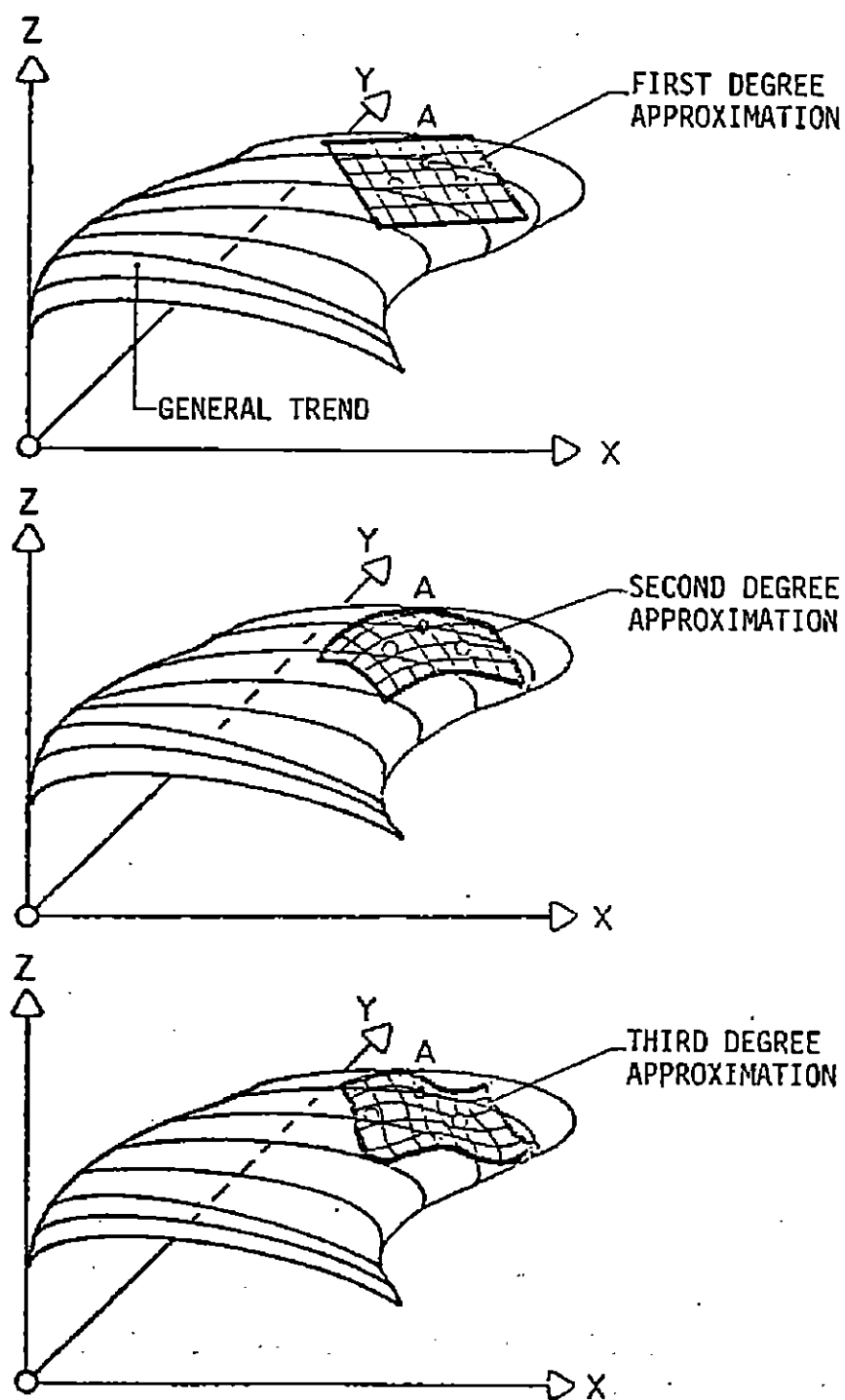


FIGURE 3 LOCAL APPROXIMATIONS OF GENERAL TRENDS OF TWO INDEPENDENT VARIABLES



where:  $\bar{Z}_B$  is the mean-value at the known data points,

$a_B$  are the unknown weight coefficients.

The random variable  $Z(x,y)$  in turn can be expressed as follows:

$$Z(x,y) = \bar{Z}(x,y) + FZ(x,y) \quad (3.3)$$

where:  $\bar{Z}(x,y)$  is the mean value, and

$FZ(x,y)$  is the fluctuating term around the mean.

This model leads to two possible groups of assumptions. They concern the first and second moments of the random variable  $Z(x,y)$ . The first group is:

$$E[Z(x,y)] = \bar{Z}(x,y) \quad (3.4)$$

$$E[Z(x_1,y_1), Z(x_2,y_2)] = \bar{Z}(x_1,y_1) \cdot \bar{Z}(x_2,y_2) + C((x_1,y_1), (x_2,y_2)) \quad (3.5)$$

where:  $\bar{Z}(x,y)$  is the mean value, and

$C(x_1,x_2)$  is the covariance of the variable  $Z(x,y)$ .

These assumptions are in many cases too restrictive and need to be replaced by more flexible ones. This can be realized by focusing on the rate of change of the random variable  $Z(x,y)$ , as follows:

$$E[Z(x_1,y_1) - Z(x_2,y_2)] = \bar{Z}(x_1,y_1) - \bar{Z}(x_2,y_2) \quad (3.6)$$

$$E[(Z(x_1,y_1) - Z(x_2,y_2))^2] = 2 \gamma((x_1,y_1), (x_2,y_2)) \quad (3.7)$$

where:  $\gamma(x_1,x_2)$  is the variogram or semivariance of the difference

$Z(x_1,y_1) - Z(x_2,y_2)$ .

Interestingly the variogram  $\gamma(d)$  and the covariance  $C(d)$  are both defined as functions of the distance between two locations and can be related by Eq. 3.8, as shown in Appendix A:

$$\gamma(d) = C(0) - C(d). \quad (3.8)$$

As shown in Figure 2,  $\gamma(d)$  and  $C(d)$  are complementary.

Both above sets of assumptions concern the stochastic model. An additional set of assumptions is needed to specify the nature of the randomness of the variable  $Z(x,y)$ , in particular its mean and its variance.

Locally at a point  $(x,y)$  the mean  $\bar{Z}(x,y)$  is approximated as a linear combination of known functions:

$$\bar{Z}(x,y) = \sum_{i=1}^k a_i f^i(x,y) \quad (3.9)$$

where:  $a_i$  are unknown weight coefficients,  
 $f^i(x,y)$  are apriori known functions locally approximating  $\bar{Z}(x,y)$ .

The global trend analysis of most real cases, Davis (2), can be represented by a polynomial equation of the first, second or third order. Therefore it can be intuitively seen that locally the trend can be best approximated by quadratic functions as shown in Figure 3. A corresponding norm can be defined for a best approximation with respect to the encountered trend.

The covariance  $C(x_1, x_2)$  can be computed from field measurements, and can be represented by the following expression:

$$C(x_1, x_2) = k e^{-\alpha d} \quad (3.10)$$

where:  $d$  = distance between  $x_1$  and  $x_2$ ,

$\alpha, k$  = fitting parameters.

In many cases the variogram  $\gamma(d)$  is more suitable than the covariance  $C(d)$  to define the distances over which, realizations of the

random variable  $Z$  are interdependent. Thus a range of maximum allowable sampling interval can be evaluated. The expressions most commonly used for  $\gamma(d)$  are:

$$\gamma(d) = \beta + 3\alpha \log(d) \quad (3.11)$$

or 
$$\gamma(d) = \alpha(d) \quad (3.12)$$

where:  $\alpha$  and  $\beta$  are fitting parameters.

The first expression is often referred to as DeWij's model, while the second as Linear model. The selection of the adequate expression for  $\gamma(d)$  is difficult since it requires a very careful treatment of the data obtained from field investigation. Matheron (5) gives a case study of such a treatment as illustrated in Figure 4. In the present study both expressions were tested.

### 3.2 Identification of the Best Estimator

Summarizing, up to this point two sets of model assumptions are given, Eqs. (3.4) and (3.5) and Eqs. (3.6) and (3.7), concerning the random variables  $Z(x,y)$ . Both these sets of assumptions can be used to identify estimators of both the random variable  $\hat{Z}(x,y)$  and its mean  $\hat{\bar{Z}}(x,y)$ .

The first model, (Eqs. (3.4) and (3.5)) is used to obtain the best estimator for the random variable  $\hat{Z}(x,y)$  itself. Both models identify the best estimators among all possible functions satisfying the hypothesis concerning the randomness of the rock media, Eq. (3.9). This is done by minimizing the variance of the estimation, Eqs. (3.5), respectively (3.7), subject to the first moment constraints, Eqs. (3.4), respectively (3.6). The Lagrange Multipliers approach is used for this constrained optimization problem. The computations carried out in Appendix B, lead to the following results:

FOR MODEL 1 concerning the estimation of the mean  $\bar{Z}(x,y)$  (from Eqs. (3.4) and (3.5)).

The method of Lagrange Multipliers leads to a set of  $n + k$  equations with  $n + k$  unknowns, namely the weight coefficients:

$$\begin{aligned} \sum_{\beta} a_{\ell}^{\beta} c_{\alpha\beta} - \sum_{\ell} \mu_{\ell} f_{\alpha}^{\ell} &= 0 ; \alpha, \beta = 1, \dots, n \\ \sum_{\alpha} a_{\ell}^{\alpha} f_{\alpha}^{\ell} - \delta_{\ell}^{\alpha} &= 0 ; \ell = 1, \dots, k \end{aligned} \quad (3.13)$$

where:  $a_{\ell}^{\beta}$  are the unknown weight coefficients,  
 $\mu_{\ell}$  are the Lagrange Multipliers,  
 $\delta_{\ell}^{\alpha}$  is the Kronecker delta.

The variance of the estimation is given by:

$$E[\hat{Z}(x,y)]^2 = \sum_{\ell} \mu_{\ell} f^{\ell}(x,y) ; \ell = 1, \dots, k \quad (3.14)$$

FOR MODEL 2 concerning the estimation of the random variable  $Z(x,y)$  (from Eqs. (3.6) and (3.7)).

Similarly the following system is obtained (Appendix B).

$$\begin{aligned} \sum_{\beta} b_{\alpha}^{\beta} \gamma_{\alpha\beta} + \sum_{\ell} \mu_{\ell} f_{\alpha}^{\ell} &= \gamma(x_{\alpha}, x) ; \alpha, \beta = 1, \dots, n \\ \sum_{\alpha} b_{\ell}^{\alpha} f_{\alpha}^{\ell} &= f^{\ell}(x) ; \ell = 1, \dots, k \\ \sum_{\alpha} b_{\ell}^{\alpha} &= 1 \end{aligned} \quad (3.15)$$

where:  $b_{\ell}^{\alpha}$  are the unknown weight coefficients,  
 $\mu_{\ell}$  are the Lagrange Multipliers.

The variance of the estimation is given by:

$$\begin{aligned} E[Z - \hat{Z}]^2 &= \sum_{\alpha} b_{\ell}^{\alpha} \gamma(x_{\alpha}, x) + \sum_{\ell} \mu_{\ell} f_{\alpha}^{\ell} ; \alpha = 1, \dots, n \\ &\ell = 1, \dots, k \end{aligned} \quad (3.16)$$

#### 4 Interfacing the Inference Model with the Analytical Model

##### 4.1 Uncertainty Analysis in the Analytical Model

As mentioned previously the analytical model is treated using the finite element technique which provides us with the transfer mechanism between a set of inputs  $\{F\}$  and a set of generally unknown outputs  $\{u\}$ . The general solution is given by the following relation in matrix form:

$$\{u\} = [K]^{-1} \{F\} \quad (4.17)$$

where:  $\{K\}$  is generally known as stiffness matrix and is defined as a function of the random variables  $Z_1, Z_2, \dots, Z_n$ ,

$\{F\}$  is the loading term.

Applying now the first order uncertainty analysis as described by Papoulis ( ), the first and second moments of the unknown vector  $\{u\}$  are obtained as follows:

##### FIRST MOMENT

$$E[\{u(Z_1, Z_2)\}] = \{u(\bar{Z}_1, \bar{Z}_2)\} + \frac{1}{2} \left[ \sigma_{Z_1}^2 \frac{\partial^2 \{u(\bar{Z}_1, \bar{Z}_2)\}}{\partial Z_1^2} + \dots \right] \quad (4.18)$$

where:  $\sigma_{Z_1}^2$  is the variance of the physical parameter  $Z_1$ .

The second part of the right hand side can be neglected being a very small quantity.

##### SECOND MOMENT

$$E[\{u(Z_1, Z_2)\}^2] = \sigma_{Z_1}^2 \left( \frac{\partial \{u(\bar{Z}_1, \bar{Z}_2)\}}{\partial Z_1} \right)^2 + \sigma_{Z_2}^2 \left( \frac{\partial \{u(\bar{Z}_1, \bar{Z}_2)\}}{\partial Z_2} \right)^2 + 2 \frac{\partial \{u(\bar{Z}_1, \bar{Z}_2)\}}{\partial Z_1} \frac{\partial \{u(\bar{Z}_1, \bar{Z}_2)\}}{\partial Z_2} \text{cov}(Z_1, Z_2). \quad (4.19)$$

The partial derivatives in Eq. (4.19) are obtained from the following system:

$$[K(Z_1, Z_2)] \frac{\partial \{u(Z_1, Z_2)\}}{\partial Z_i} = \frac{\partial \{F\}}{\partial Z_i} - \frac{\partial [K(Z_1, Z_2)]}{\partial Z_i} \{u\};$$

$$i = 1, 2 \quad (4.20)$$

The solution is obtained by the classical finite element methodology, ( ).

#### 4.2 Coupling of the Inference Model with the Analytical Uncertainty Model

The estimates of the spatial mean  $\hat{Z}_1(x, y)$ ,  $\hat{Z}_2(x, y)$  and the variances  $\sigma_{Z_1}(x, y)$ ,  $\sigma_{Z_2}(x, y)$  provided by the inference model are substituted in the statistical relations of the dependent random variable  $\{u\}$ , Eqs. (4.18) and (4.19). Thus the first and second moments, as well as the coefficient of variation of  $\{u\}$  can be evaluated. The coefficient of variation in particular being an essential statistical property, can be used to evaluate the performance of the analytical model.

For testing purposes, an example was treated borrowed from the field of underground confined flow, as shown in section 6. The output of the analysis provides the first and second moments of the unknown hydraulic head  $\{u\}$ , at any specified location of the domain under consideration.

#### 5 Description of the Algorithm

The above described procedure is summarized by the computation steps of the flow chart of Fig. 5. The geometric domain under investigation is divided using a rectangular mesh common for the Inference

Model and the analytical model (Finite element mesh). The computations are carried out at each node using a number among the known realizations  $Z_g(x,y)$ . Therefore a zone of influence, characteristic of the media are depending on the covariance  $C(x_1, x_2)$  and/or the variogram  $\gamma(d)$  is defined at each node of the mesh. For computational efficiency, 'n' known points are selected within this zone, to determine the estimation of the random variable  $Z(x,y)$  at that particular node. A polynomial comporting k terms, approximates the trend of  $Z(x,y)$  in the neighborhood of the node at hand. Thus, at every node  $(x,y)$ , a system of  $n + k$  equations permits to determine the estimator  $\hat{Z}(x,y)$ . The outcome of the procedure depends on the number of known points and the density of the provided information  $Z_g(x,y)$ .

In case of insufficient information in the domain of interest, the original assumptions are violated. However, in this case the values of variance of the estimation indicate the poor performance of the inference and give the exact location where more information is needed.

The flow chart of Figure 5 gives the sequence in which the computations are performed by program INFMOD. The mean values of the physical parameters evaluated by the inference model INFMOD are introduced at each node of the finite element mesh. Thereafter, the computations are performed in a conventional way taking into account the prescribed boundary conditions. The mean values of the unknown vector  $\{u\}$  are determined, as well as the vectors  $\frac{\partial \{u\}}{\partial Z_i}$ ,  $i = 1, 2$ . Finally the variance of the unknown vector  $\{u\}$  is computed at each node of the mesh.

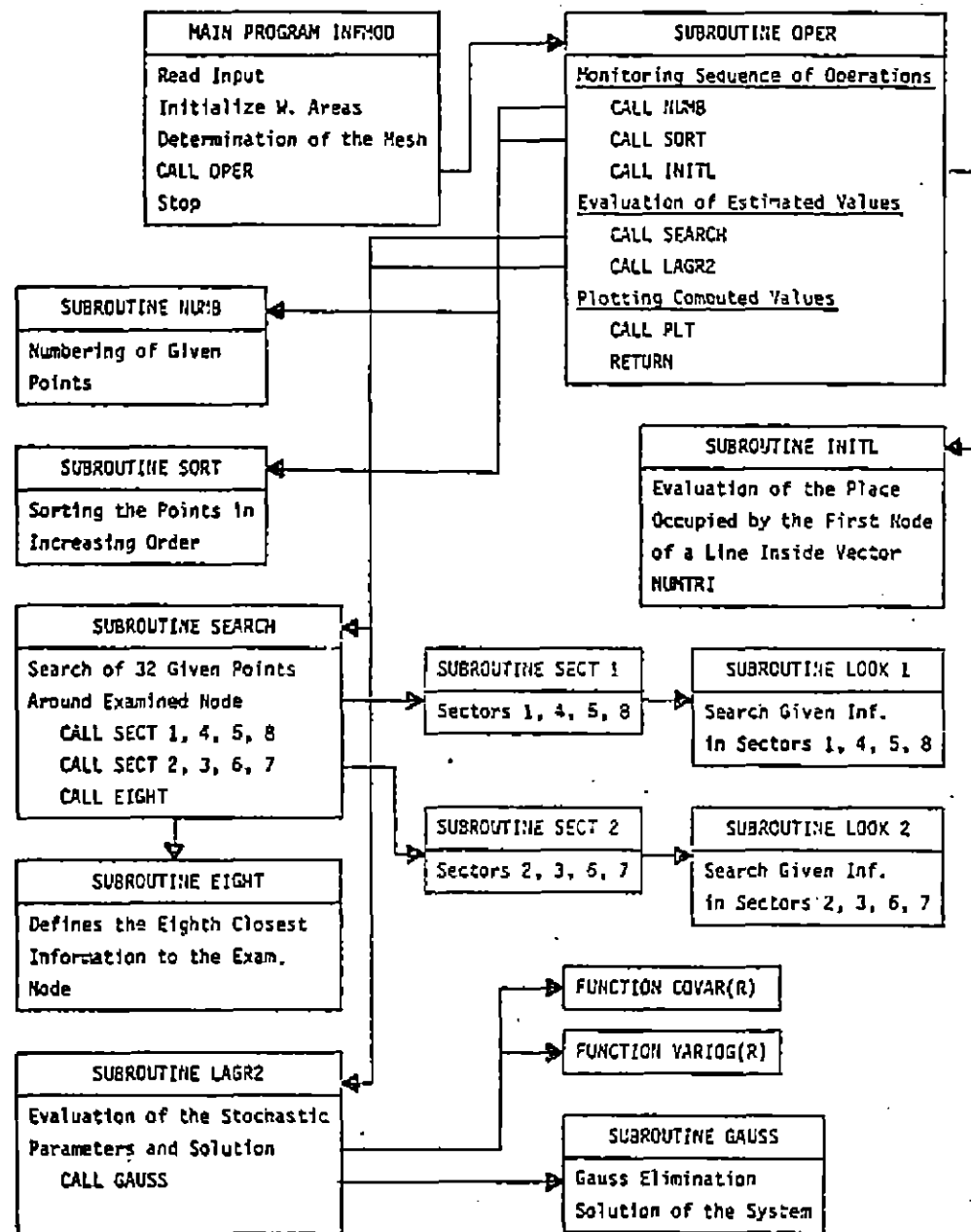


FIGURE 5 FLOW CHART OF PROGRAM INFMOD (INFERENCE MODEL)



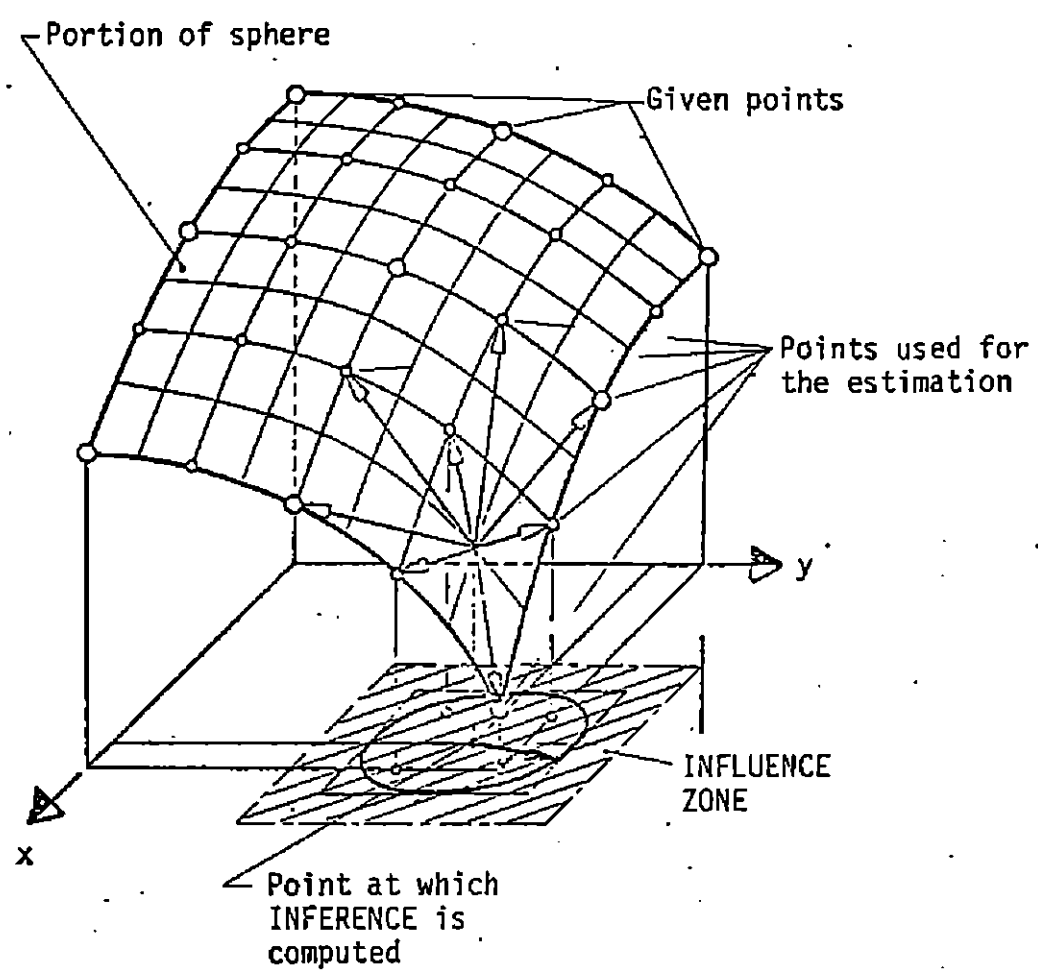


FIGURE - 6 ILLUSTRATION OF PROBLEM 1 (SPHERICAL TREND)

As expected, increasing amount of information, produces a smaller variance of the estimates. The case of largest variance is reported in Table 1.

It is to be mentioned that the maximum error obtained is less than 0.1 percent and that the estimated first and second moments are symmetrically distributed with respect to the existing plane of symmetry.

Another interesting feature is that the spatial distribution of the error is similar to that of the variance, justifying the use of the estimated variance as an indicator of the error.

## 2. Tests of the statistical assumptions

These tests concerned (a) the assumptions on the apriori known function  $f(y)$  characterizing the behavior of the mean  $\bar{Z}(x,y)$ , and (b) the assumption on the general form of the variogram obtained either from site investigation or considered apriori. The tests were performed on the square 400 x 400m of Problem 1 and over a region 50 x 20m of the flow problem (Problem 2).

Several functions  $f(x,y)$  were tested in the above mentioned regions and the most satisfactory results from the point of view of both accuracy and efficiency were obtained for the following quadratic function:

$$f(x,y) = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy \quad (5.21)$$

This form was also suggested from a trend analysis (sect. 2), applied to the data of the flow problem as illustrated in Figure 8.

The following expressions for the variogram were studied:

$$\gamma(d) = \alpha \log |d| \quad (5.22)$$

$$\gamma(d) = \alpha |d| \quad (5.23)$$

TABLE 1 RESULTS OF INFERENCE MODEL FOR PROBLEM 1

NUMBER OF GIVEN INFORMATION	TYPE OF MODEL	MAX ERROR	EXPECTED VALUE	MAX VARIANCE	MAX COEFFICIENT OF VARIATION
9 points	MODEL 1	0.056	3.18	.81	0.28
	MODEL 2	0.049	3.45	1.08	0.30
25 points	MODEL 1	0.011	3.33	.35	0.17
	MODEL 2	0.0047	3.61	0.569	0.21
81 points	MODEL 1	0.002	3.40	0.21	0.135
	MODEL 2	0.0007	3.62	0.249	0.14

TABLE 2 RESULTS OF INFERENCE MODEL FOR PROBLEM 2

COORDINATES		ESTIMATED MEAN	ESTIMATED VARIANCE	COEFFICIENT OF VARIATION
x	y	$10^{-3}$ cm/sec	$10^{-5}$ cm/sec	$10^{-2}$
5.000	5.000	.013	.002	3.651
10.000	5.000	.008	.002	6.137
15.000	5.000	.002	.003	32.673
20.000	5.000	.015	.001	2.167
25.000	5.000	.027	.002	1.813
30.000	5.000	.008	.001	4.449
35.000	5.000	.030	.004	2.041
40.000	5.000	.026	.000	.805
45.000	5.000	.018	.001	1.698
50.000	5.000	.171	.071	1.559
0	10.000	.097	.029	1.736
5.000	10.000	.026	.001	.905
10.000	10.000	.012	.002	3.290
15.000	10.000	.080	.000	.000
20.000	10.000	.053	.004	1.198
25.000	10.000	.061	.001	.603
30.000	10.000	.065	.010	1.534
35.000	10.000	.003	.002	13.735
40.000	10.000	.025	.004	2.696
45.000	10.000	.091	.003	.590
50.000	10.000	.121	.262	4.229
0	15.000	.503	.026	.323
5.000	15.000	.335	.002	.119
10.000	15.000	.254	.004	.262
15.000	15.000	.331	.001	.069
20.000	15.000	.460	.003	.121
25.000	15.000	.454	.001	.053
30.000	15.000	.418	.003	.120
35.000	15.000	.502	.001	.066
40.000	15.000	.315	.002	.133
45.000	15.000	.542	.002	.079
50.000	15.000	.599	.082	.479

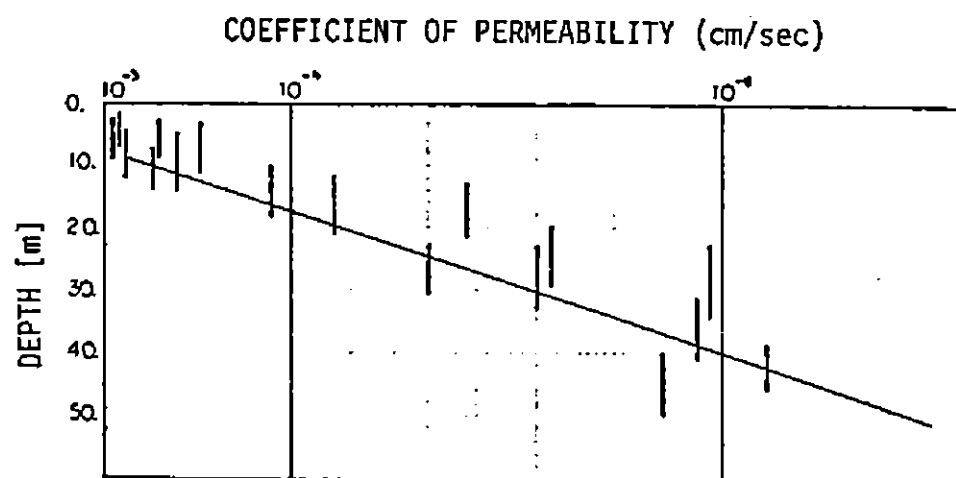


FIGURE 8 GENERAL TREND FUNCTION OF THE PERMEABILITY

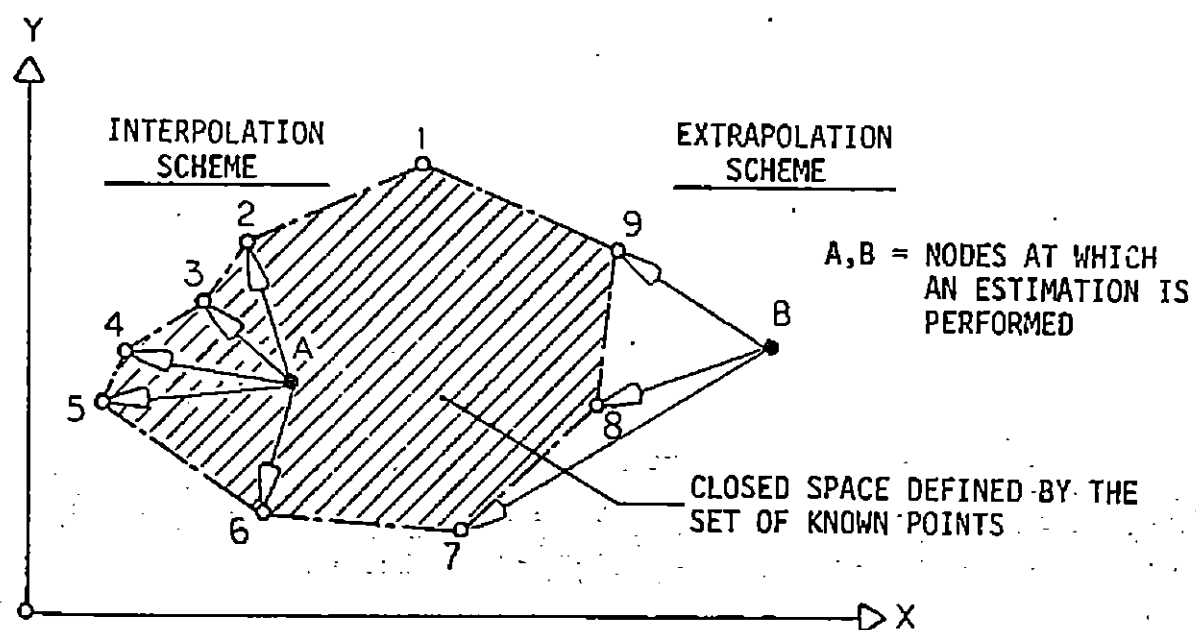


FIGURE 9 TOPOLOGICAL CONSIDERATIONS FOR THE ESTIMATION NODE

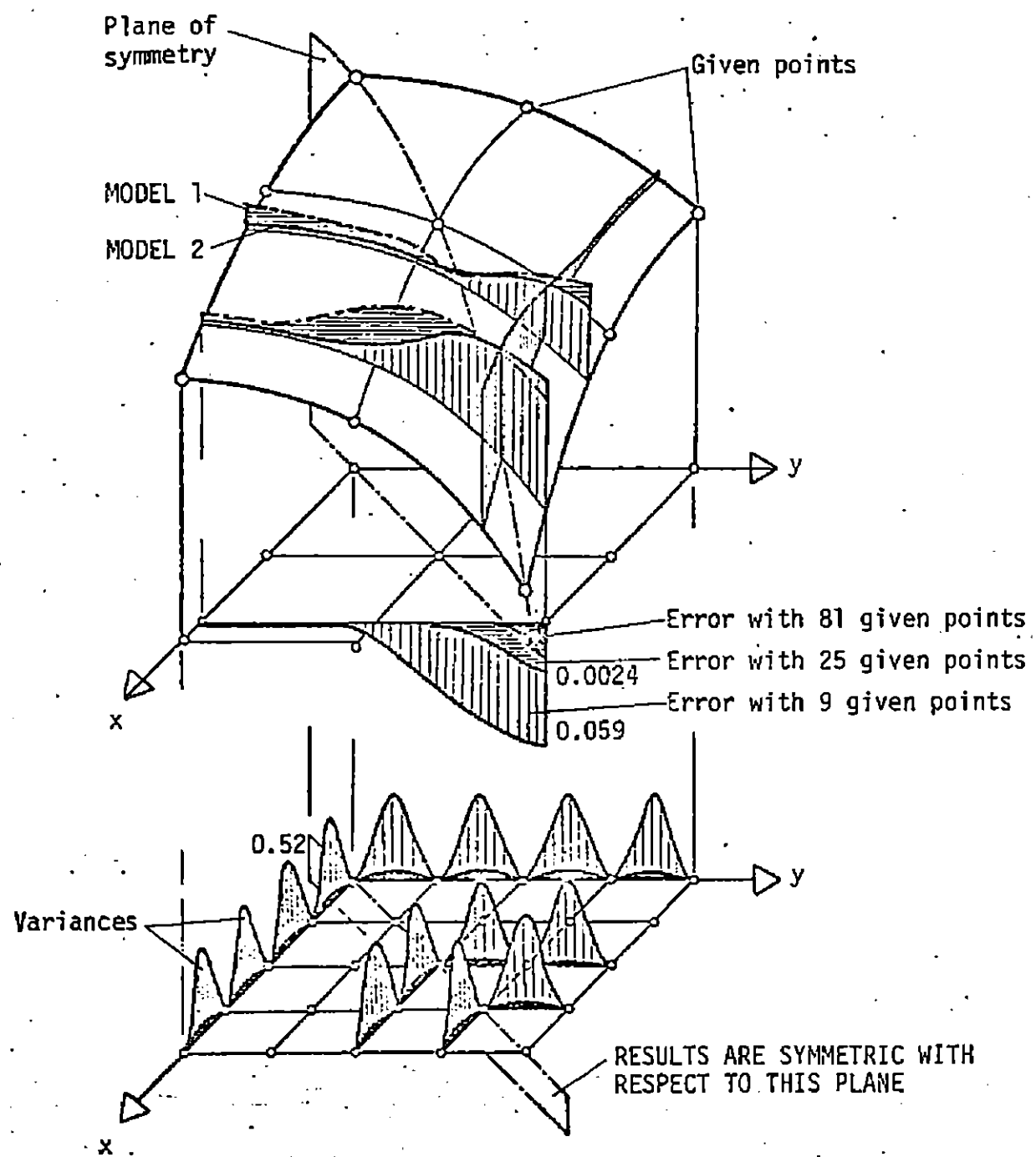


FIGURE 7 ESTIMATED STATISTICAL CHARACTERISTICS OF PROBLEM 1

## 6 Example of Application and Discussion

As said in section 4 the example of application is borrowed from the field of underground confined flow. Two groups of tests are conducted to explore the limitations and the applicability of the general algorithm. One group concerns the inference model alone, while the other deals with the performance of the finite element model.

### FIRST GROUP

The three different tests related to the Inference model, in connection with two different problems, produced the following results:

#### 1. Tests of the statistical convergence

This test was conducted under the assumption of statistical isotropy and for a uniform spatial distribution of the given information points. More specifically in Problem 1 a domain defined by a square mesh of 400 x 400m was examined and the random variable  $Z(x,y)$  was assumed to possess a realization lying on a portion of a sphere over this domain, as shown in Figure 6. The domain was divided into squares of 25 x 25m having 289 nodes where the computations were performed. The given information was located first on nine points as shown in Figure 6, then on twenty-five points and finally on eighty-one points.

The estimation at the 288 nodes of the mesh was performed by both Inference models defined previously (Sect. 3). Interestingly enough in the region of great variability of the random variable  $Z(x,y)$  the two models show a good agreement in their estimation, as illustrated in Figure 7.

## SECOND GROUP

The following tests, related to the coupling of the uncertainty analysis using F.E.M. and the inference model were performed:

1. Tests for the convergence of the statistical characteristics of  $\{u\}$  as a function of the size of the element's mesh.

For the First Moment this was done automatically through a mesh generation subroutine. For the Second Moment, a denser mesh was considered only around the flow barriers, where the hydraulic gradient was important.

A satisfactory convergence was observed, as shown in Table 2.

2. Tests of the effect of the boundary conditions on the statistical characteristics of  $\{u\}$ .

Several differential hydraulic heads were considered and their results plotted in Figure 12. They display expected responses, namely as the differential head increases, the variance of the unknown quantity  $\{u\}$  increases also, showing that the flow becomes more variable reaching eventually a state of nonlaminar flow. Interestingly enough the variances seem to be directly related to the hydraulic gradient, Figure 13.

### 7 Remarks on the Applicability of the Method

Becker, Hazen and Scott (1) have produced statistical evidence that data selected from random samples produce more realistic information. Attention then should be given to the randomness of the sample points, as well as the uniformity of the sample volume. The Inference model could be used to advantage here as follows:

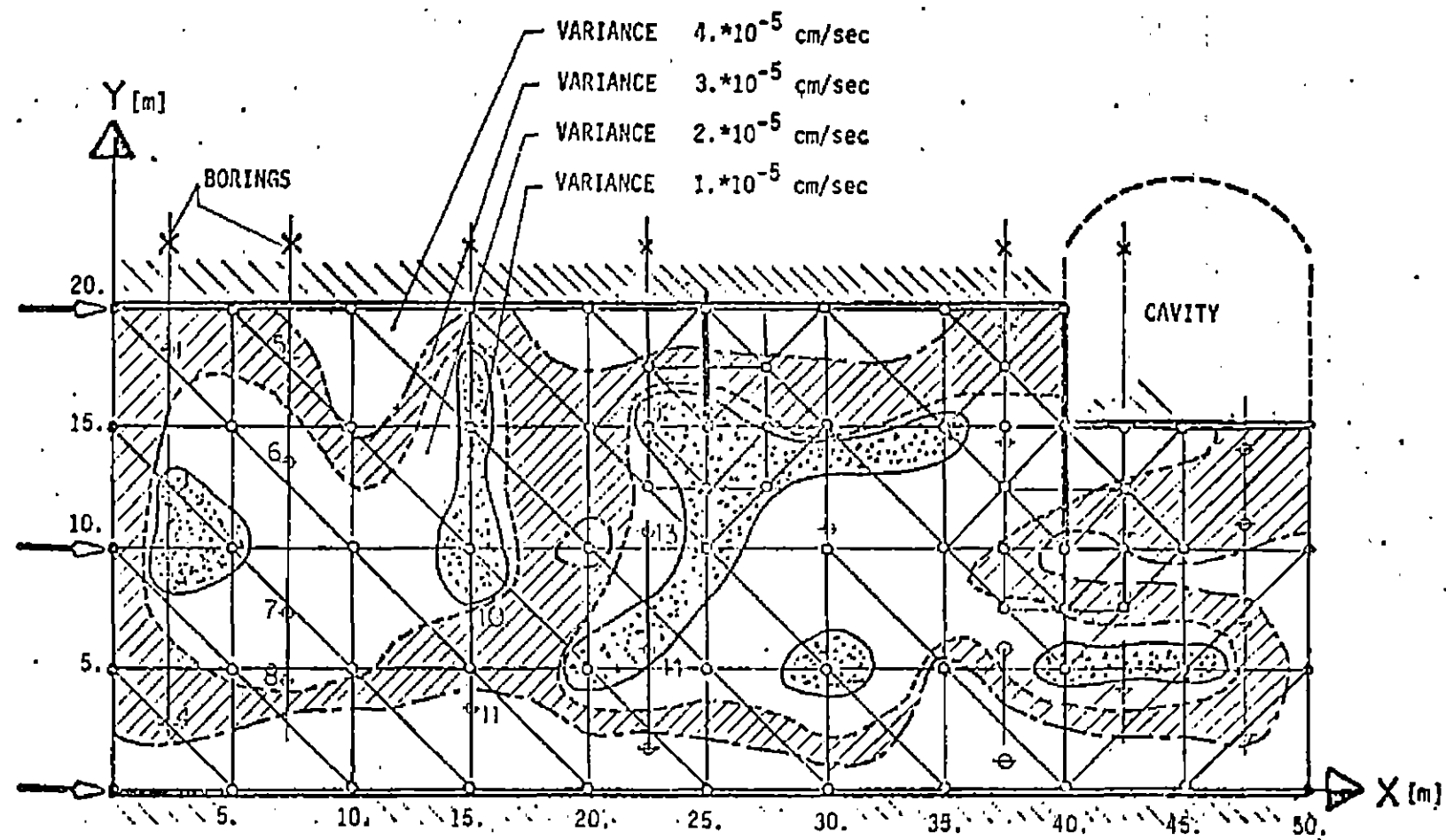


FIGURE 11 ESTIMATED STATISTICAL CHARACTERISTICS OF PROBLEM 2 (HORIZONTAL FLOW)



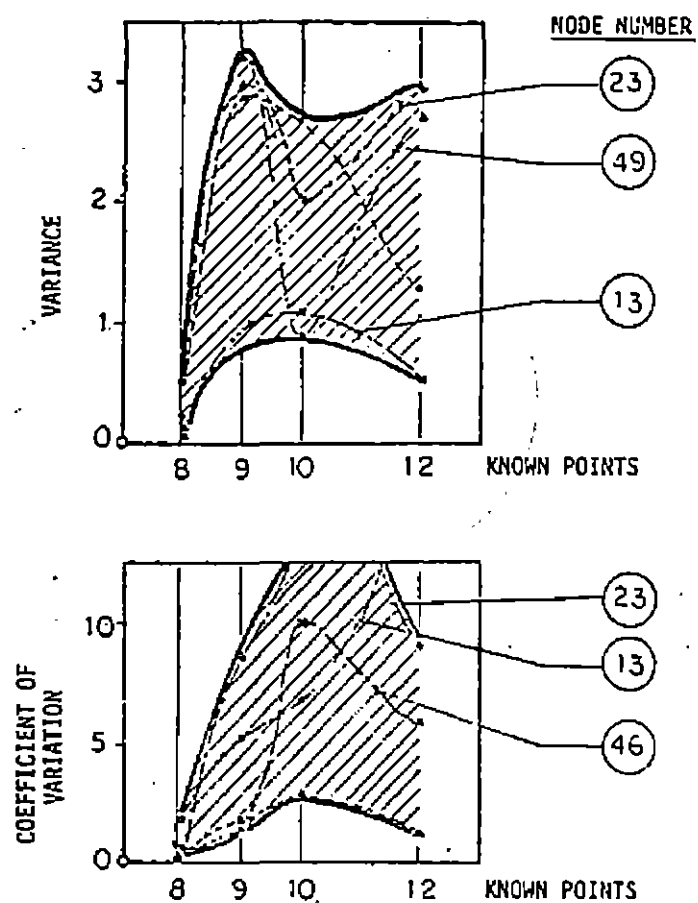


FIGURE 10 ESTIMATED VARIANCES VS. THE NUMBER OF KNOWN POINTS

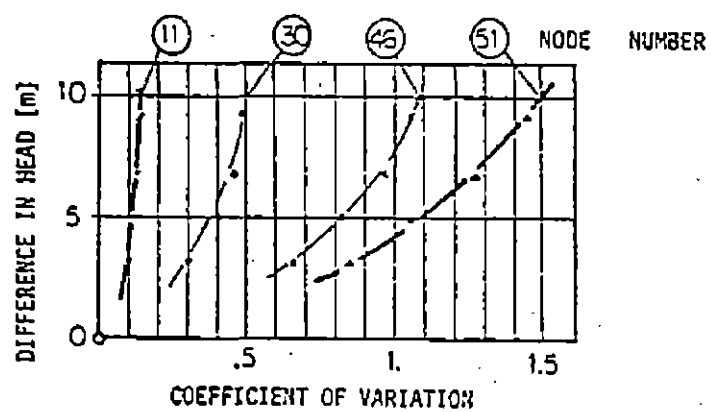


FIGURE 12 EFFECT OF BOUNDARY CONDITIONS ON ESTIMATED STATISTICAL VALUES

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### 1. Randomness of data points

In the Inference model, the data randomness is automatically tested through the use of the functions describing the statistical constraints, Eqs. 13 and 15. Indeed, if the provided information does not follow a random pattern, numerical instability is induced which shows that complementary information is needed.

### 2. Uniform sample volume

It is recognized that a basic requirement for the successful use of statistics in sampling is the uniformity of the sample volume. The Inference model accounts for this size effect through the use of the notion of variogram, (sect. 2 ). The variogram permits to determine the range of significant statistical inference around a given point. On the other hand it was seen that a one meter boring was sufficient to produce a good statistical description of the physical characteristics of the rock. Therefore, it is proposed here, to use the average values obtained from one meter of borings as the input information to the Inference model, as illustrated in Figure 14.

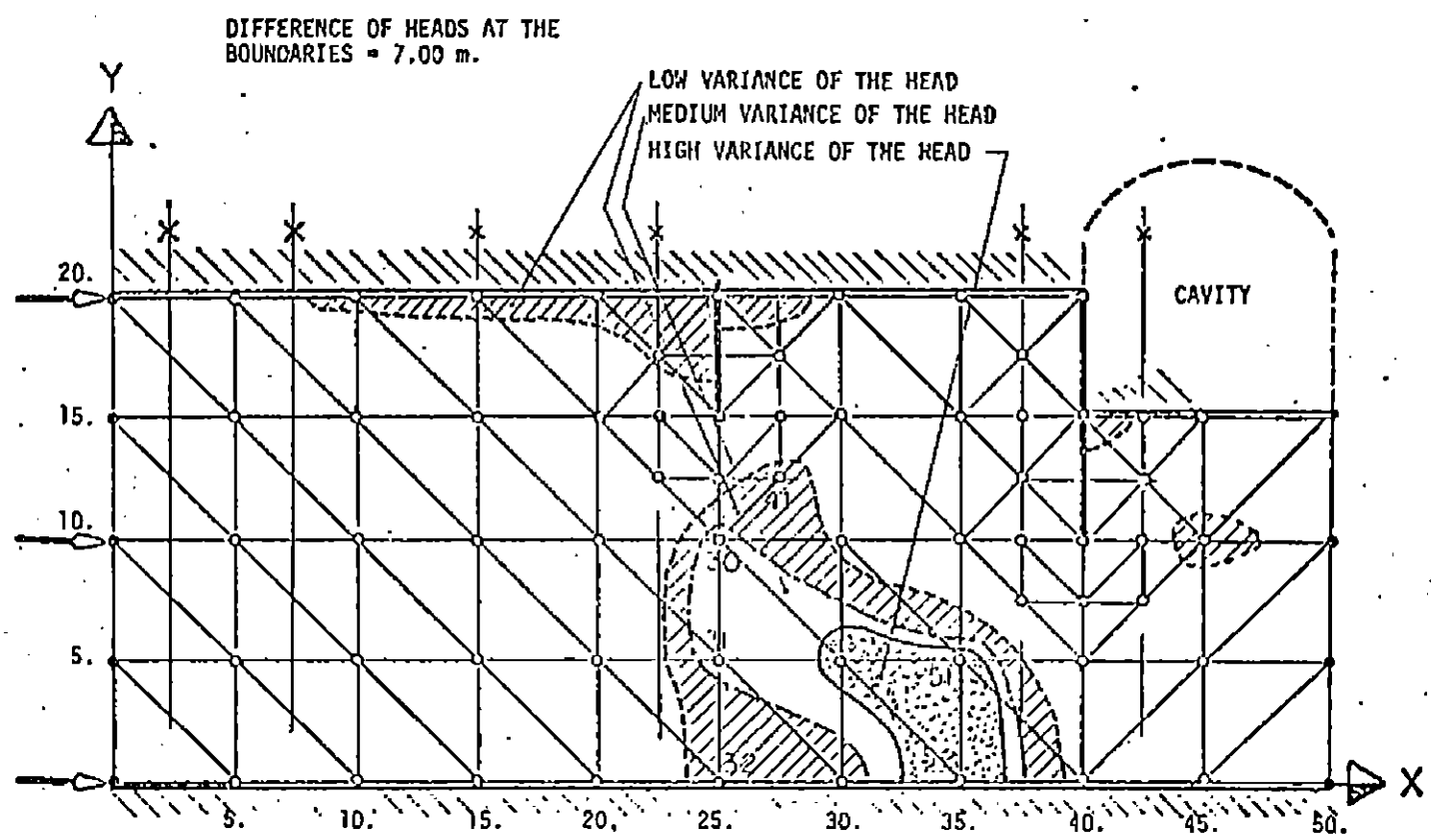


FIGURE 13 RESULTS OF THE INFERENCE MODEL COUPLED WITH THE F.E.M. (HORIZONTAL FLOW)

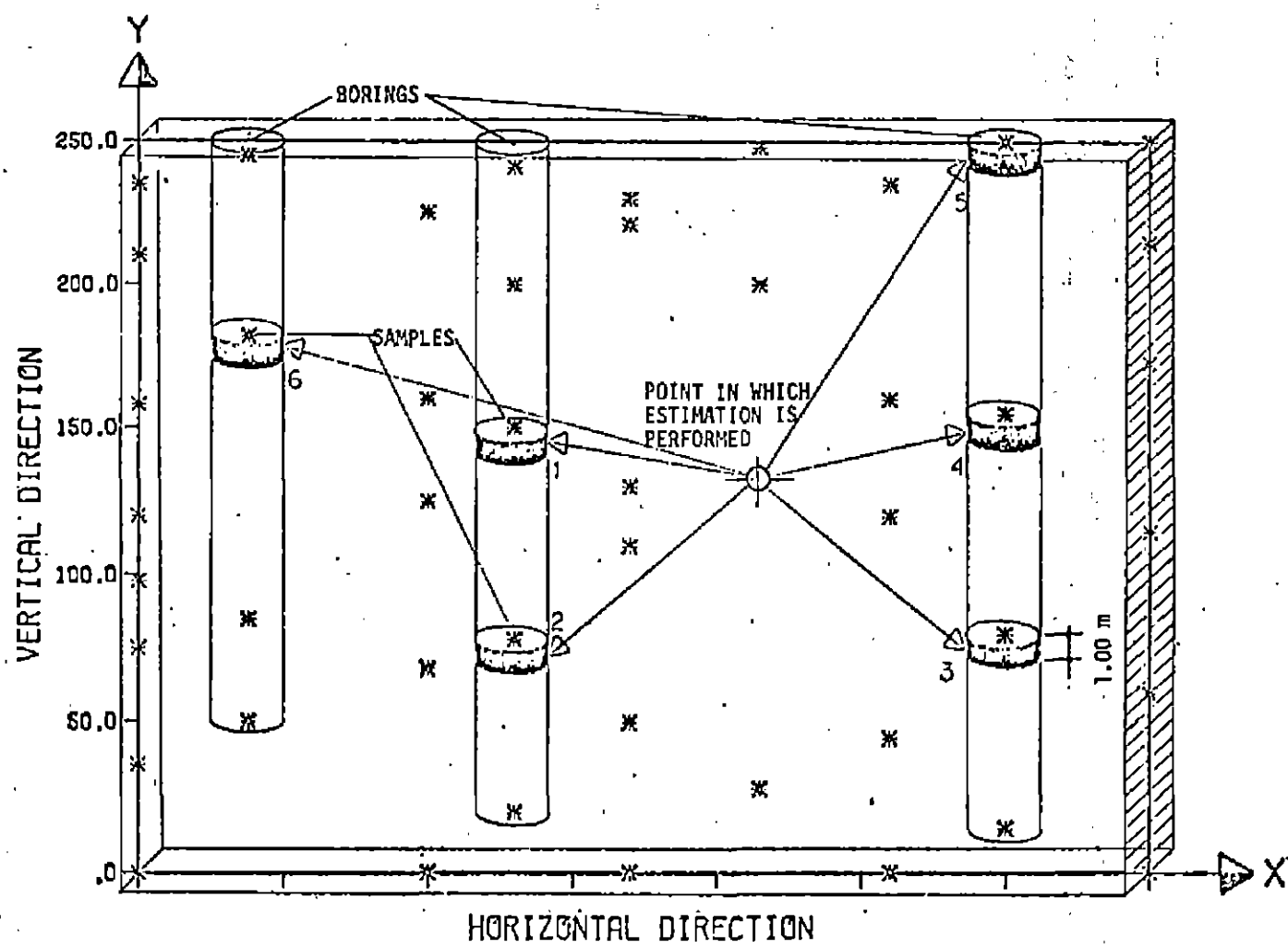


FIGURE 14 ILLUSTRATION OF THE ESTIMATION PROCEDURES

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## Appendix A

### Computations Related to the Inference Model

#### 1. Relationship between Covariance and Variogram

Using the general expression of the second statistical moment one can obtain

$$\begin{aligned} E \left[ \left( (Z_x - Z_y) - (\hat{Z}_x - \hat{Z}_y) \right)^2 \right] &= E \left[ (Z_x - Z_y)^2 \right] + E \left[ (\hat{Z}_x - \hat{Z}_y)^2 \right] - \\ &\quad - 2E \left[ (Z_x - Z_y)(\hat{Z}_x - \hat{Z}_y) \right] = \\ &= E(Z_x - Z_y) E(Z_x - Z_y) + C(Z_x - Z_y, Z_x - Z_y) + E(\hat{Z}_x - \hat{Z}_y) E(\hat{Z}_x - \hat{Z}_y) \\ &\quad + C(\hat{Z}_x - \hat{Z}_y, \hat{Z}_x - \hat{Z}_y) - 2E(\hat{Z}_x - \hat{Z}_y) E(Z_x - Z_y) - \\ &\quad - 2C(Z_x - Z_y, \hat{Z}_x - \hat{Z}_y) \end{aligned}$$

where:  $C$  = the covariance of variables  $Z$  and  $\hat{Z}$

which after a few simplifications leads to

$$\begin{aligned} E \left[ (Z_x - Z_y, \hat{Z}_x - \hat{Z}_y)^2 \right] &= C(Z_x - Z_y, Z_x - Z_y) + \\ &\quad C(\hat{Z}_x - \hat{Z}_y, \hat{Z}_x - \hat{Z}_y) - 2C(Z_x - Z_y, \hat{Z}_x - \hat{Z}_y) = \\ &= 2C(Z_x - Z_y, Z_x - Z_y) - 2C(Z_x - Z_y, \hat{Z}_x - \hat{Z}_y) \end{aligned}$$

Making use of the general statistical assumptions given in section (A.3)

we obtain

$$E \left[ \frac{\left( (Z_x - Z_y) - (\hat{Z}_x - \hat{Z}_y) \right)^2}{2} \right] = C(0) - C(Z_x - Z_y, \hat{Z}_x - \hat{Z}_y)$$

which leads to the expression of the variogram  $\gamma$

$$\gamma[(Z_x - Z_y), (\hat{Z}_x - \hat{Z}_y)] = C(0) - C(Z_x - Z_y, \hat{Z}_x - \hat{Z}_y)$$

or

$$\gamma(d) = C(0) - C(d) \quad (A.1)$$

## 2. Optimization Using Lagrange Multipliers

The problem consists to minimize  $E[(z-\hat{z})^2]$  the variance with the constraint  $E[z(x,y)] - E[\hat{z}(x,y)] = 0$ .

Using the assumptions introduced in section (5.3.1) we obtain

$$E[(z-\hat{z})^2] = C(z,z) + \sum_{\alpha} b_{\alpha} \sum_{\beta} b_{\beta} [C(z_{\alpha}, z_{\beta})] - 2 \sum_{\alpha} b_{\alpha} C(z, z_{\alpha}) \quad (A.2)$$

and

$$\begin{aligned} E[z] - E[\hat{z}] &= \sum_{\ell} a_{\ell} f^{\ell} - \sum_{\ell} a_{\ell} \left[ \sum_{\alpha} b^{\alpha} f_{\alpha}^{\ell} \right] = \\ &= \sum_{\ell} a_{\ell} \left[ f^{\ell} - \sum_{\alpha} b^{\alpha} f_{\alpha}^{\ell} \right] = 0; \end{aligned} \quad (A.3)$$

$\alpha = 1, \dots, n \quad \ell = 1, \dots, k$

The minimization of the variance will be obtained using the method of Lagrange multipliers as follows.

The Lagrange function being

$$\begin{aligned} L &= C(z,z) - 2 \sum_{\alpha} b_{\alpha} C(z, z_{\alpha}) + \sum_{\alpha, \beta} b^{\alpha} b^{\beta} C(z_{\alpha}, z_{\beta}) - \\ &- \sum_{\ell} (\mu_{\ell}) \left[ f^{\ell} - \sum_{\alpha} b^{\alpha} f_{\alpha}^{\ell} \right]; \end{aligned} \quad (A.4)$$

$\alpha, \beta = 1, \dots, n \quad \ell = 1, \dots, k$

The conditions to obtain the minimum are

$$\frac{\partial L}{\partial b^{\alpha}} = 0 \quad \text{for all } b \text{ 's} \quad (A.5)$$

$$\frac{\partial L}{\partial \mu_{\ell}} = 0 \quad \text{for all } \mu \text{ 's}$$

The unknowns being  $b$  's and  $\mu$  's we obtain a linear system of  $n+k$  equations.

The differentiation of  $L$  with respect to  $b^\alpha$  and  $M_\ell$  gives:

First with respect to  $b^\alpha$ .

$$-2 C(z, z_\alpha) + \sum_{\beta} b^\beta C(z_\alpha, z_\beta) + \sum_{\ell} \mu_\ell f_\alpha^\ell = 0 \quad \forall \alpha = 1, n \quad (A.7)$$

Second with respect to  $\mu_\ell$ .

$$-f^\ell + \sum_{\alpha} b^\alpha f_\alpha^\ell = 0 \quad \forall \ell = 1, k \quad (A.8)$$

The system then can be written as

$$\begin{aligned} \sum_{\beta} b^\beta C(z_\alpha, z_\beta) + \sum_{\ell} \mu_\ell f_\alpha^\ell &= 2 C(z_\alpha, z) \\ \sum_{\alpha} b^\alpha f_\alpha^\ell &= f^\ell \end{aligned} \quad (A.9)$$

The covariances  $c(z_\alpha, z_\beta)$  and  $c(z_\alpha, z)$  are obtained from the following relations:

$$C(z_\alpha, z) = -\gamma(Fz_\alpha - Fz) \text{ and } C(z_\alpha, z_\beta) = -\gamma(Fz_\alpha - Fz_\beta)$$

Then the linear system of equations becomes

$$\begin{aligned} \sum_{\beta} b^\beta \gamma(z_\alpha - z_\beta) + \sum_{\ell} \mu_\ell f_\alpha^\ell &= 2 \gamma(z_\alpha - z) \\ \sum_{\beta} b^\beta f^\ell(z_\beta) &= f^\ell(z) \end{aligned} \quad (A.10)$$

where the  $b$ 's and  $\mu$ 's are the unknown quantities. Therefore, solving this system the estimator of the variable is defined by:

$$\hat{Z}(x, y) = \sum_{\beta} b^\beta z_\beta \quad (A.11)$$

and the variance of the estimate is

$$\sigma_{\hat{Z}}^2 = \sum_{\beta} b^\beta \gamma(z_\beta - z) + \sum_{\ell} \mu_\ell f^\ell(z) \quad (A.12)$$



```

PROGRAM INFMDD (INPUT,OUTPUT,PLOT,TAPE1)
-----
                                VERSION DN1677

THIS PROGRAM PERFORMS AN INFERENCE MODEL ANALYSIS
BASED ON A MOVING AVERAGE TECHNIQUE

INPUT DATA

    DELTA    = MESH INTERVAL
    NPL      = 1      PLOT REQUIRED
              = 0      WITHOUT PLOT
    N        = INITIAL PARAMETER OF VARIOGRAM
    CO       = INITIAL DISCONTINUITY

    NPGIV    = TOTAL NUMBER OF GIVEN POINTS
    X(I),Y(I)= COORDINATES OF GIVEN POINT (I)
    Z(I)     = SPECIFIED PHYSICAL PARAMETER AT LOCATION (I)

OUTPUT OF PROGRAM

    AT LOCATIONS SPECIFIED BY THE NODES OF THE GIVEN
    MESH, THE FIRST AND SECOND STATISTICAL MOMENTS
    OF THE PHYSICAL PARAMETER UNDER INVESTIGATION
    ARE ESTIMATED
-----

COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)
1,NPDIITE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y
2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)
COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG(8),DIST
11(8),LNK,KNY,K3,NBEQ
COMMON /B1/ M,CO,N,MNM
COMMON /B3B/ B3(20,20),CV(20,20)

READ INPUT DATA
=====

101 READ 110, TESTS
CONTINUE
READ 114, DELTA,NPL
READ 115, M,CO
PRINT 111

READ 113, NPGIV
IF (NPGIV.EQ.0) GO TO 109
DO 102 I=1,NPGIV

    READ 116, IDNC,X(I),Y(I)
102 CONTINUE

    DO 103 I=1,NPGIV
103 READ 116, IDN,Z(I)

    DO 104 I=1,NPGIV
104 PRINT 115, X(I),Y(I),Z(I)

PRINT 111

CALL GRAPH (1,NPGIV,0,X,Y)

=====

INITIALIZE
=====

DO 105 I=1,160

```

NUM(I)=0	A	700
NUMTRI(I,1)=0	A	710
NUMTRI(I,2)=0	A	720
NG(I)=0	A	730
ND(I)=0	A	740
NPOINTE(I)=0	A	750
NUMEL(I)=0	A	760
105 CONTINUE	A	770
MPLA(1)=0	A	780
MPLA(2)=0	A	790
DO 106 J=1,32	A	800
NHETG(J)=0	A	810
106 DIST(J)=0.	A	820
DO 107 I=1,4	A	830
XENT(I)=0.	A	840
YENT(I)=0.	A	850
DO 107 J=1,8	A	860
NTR(I,J)=0	A	870
NHEIG1(J)=0	A	880
DIST1(J)=0.	A	890
107 CONTINUE	A	900
C	A	910
C	A	920
C	A	930
C	A	940
C	A	950
C	A	960
XMAX=X(1)	A	970
XMIN=XMAX	A	980
YMAX=Y(1)	A	990
YMIN=YMAX	A	1000
DO 108 I=1,NPGIV	A	1010
X1=X(I)	A	1020
Y1=Y(I)	A	1030
IF (X1.LE.XMIN) XMIN=X1	A	1040
IF (Y1.LE.YMIN) YMIN=Y1	A	1050
IF (X1.GE.XMAX) XMAX=X1	A	1060
IF (Y1.GE.YMAX) YMAX=Y1	A	1070
108 CONTINUE	A	1080
C	A	1090
PRINT 112, XMIN,XMAX,YMIN,YMAX	A	1100
C	A	1110
C	A	1120
C	A	1130
CALL OPER (XAH)	A	1140
C	A	1150
C	A	1160
C	A	1170
C	A	1180
IF (NPL. EQ. 0) GO TO 999	A	1190
C	A	1200
CALL PLT ( XAH,XMAX,XMIN,NINTX,NINTY )	A	1210
C	A	1220
C	A	1230
GO TO 101	A	1240
C	A	1250
C	A	1260
109 STOP	A	1270
C	A	1280
110 FORMAT (A3)	A	1290
111 FORMAT (1H1)	A	1300
112 FORMAT (/2X,8F10.4)	A	1310
113 FORMAT (8I10)	A	1320
114 FORMAT (F10.0,I2)	A	1330
115 FORMAT (2F10.3,F15.4)	A	1340
116 FORMAT (I2,2F10.3)	A	1350
C	A	1360
END	A	1370
SUBROUTINE OPER (XAH)	B	10
C	B	20
C	B	30

C		B	750
	101 CONTINUE	B	760
	102 CONTINUE	B	770
C		B	780
	RETURN	B	790
C		B	800
	103 FORMAT (5X, 7H X ,2X, 7H Y ,2X, 1H*,2X, 10HESTIMATED ,2	B	810
	1X, 10HESTIMATED ,2X, 1H*,2X, 12HCOEFFICIENT )	B	820
	104 FORMAT (5X, 7H ,2X, 7H ,2X, 1H*,2X, 10HMEAN ,2	B	830
	1X, 10HVARIANCE ,2X, 1H*,2X, 12HOF VARIATION)	B	840
	105 FORMAT (5X, 61H=====	B	850
	1=====,//)	B	860
	106 FORMAT (5X,F7.3,2X,F7.3,2X, 1H*,2X,F10.3,2X,F10.3,2X, 1H*,2X,F12	B	870
	1.3)	B	880
C		B	890
	END	B	900
	SUBROUTINE PLT (XAH,XMAX,XMIN,NINTX,NINTY)	C	10
C		C	20
C	*****	C	30
C		C	40
C	THIS SUBROUTINE PLOTS THE ESTIMATED MEAN VALUES	C	50
C		C	60
C		C	70
	DIMENSION XAH(20), YAH(20), WAH(300,4), DAH(20,20,2), TL(8)	C	80
	COMMON /BBB/ BB(20,20),CV(20,20)	C	90
C		C	100
C		C	110
C	PLOTTING THE COMPUTED VALUES	C	120
C	=====	C	130
C		C	140
	MAXDIM=300	C	150
	DO 101 I=1,NINTY	C	160
	DO 101 J=1,NINTX	C	170
	DAH(I,J,1)=BB(I,J)	C	180
	DAH(I,J,2)=CV(I,J)	C	190
101	CONTINUE	C	200
C		C	210
	XLNTH=6.	C	220
	YLNTH=4.	C	230
C		C	240
	DELTA X = XMAX / XLNTH	C	250
C		C	260
	DELTA X=2.	C	270
	N1=NINTX	C	280
	NFNS=NINTY	C	290
C		C	300
	CALL PLOTS	C	310
C		C	320
	DO 104 L=1,2	C	330
	NNG=0	C	340
	READ 105, TL	C	350
	YMAH=0.	C	360
	DELTAY=2.	C	370
C		C	380
	DO 103 I=1,NFNS	C	390
	DO 102 J=1,NINTX	C	400
	YAH(J)=DAH(I,J,L)	C	410
102	YAH(J)=DAH(I,J,L)	C	420
C		C	430
C		C	440
C		C	450
	CALL HIDE (XAH,YAH,WAH(1,1),WAH(1,2),WAH(1,3),WAH(1,4),NNG,M	C	460
1	AXDIM,N1,NFNS,TL,XLNTH,YLNTH,XMIN,DELTA X,YMAH,DELTAY)	C	470
C		C	480
C		C	490
C		C	500
103	CONTINUE	C	510
	CALL PLOT (14.,-2.,-3)	C	520
104	CONTINUE	C	530
	CALL PLOT (0,0,999)	C	540
	RETURN	C	550

C	105 FORMAT (SA10)	C	560
C	END	C	570
C	SUBROUTINE NUMB	C	580
C		C	590
C	-----	D	10
C	NUMBERING OF GIVEN POINTS	D	20
C	-----	D	30
C		D	40
C		D	50
C	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	D	60
	1,NP(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	D	70
	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	D	80
	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIGL(8),DIST	D	90
	11(8),LNK,KNY,K3,NBEQ	D	100
C	NINTX=(XMAX-XMIN)/DELTA+1	D	110
	XMAX=XMIN+NINTX*DELTA	D	120
	XMAX1=XMAX-XMIN	D	130
	NNDX=NINTX+1	D	140
	NINTY=(YMAX-YMIN)/DELTA+1	D	150
	YMAX=YMIN+NINTY*DELTA	D	160
	YMAX1=YMAX-YMIN	D	170
	NNDY=NINTY+1	D	180
C	PRINT 100, NINTX, NINTY	D	190
C	100 FORMAT( /2X,10I10)	D	200
C		D	210
C	L4=4*NINTX	D	220
	LY=NINTY-1	D	230
	DO 104 I=1,NPGIV	D	240
	XI=X(I)-XMIN	D	250
	YI=Y(I)-YMIN	D	260
	IX=XI/DELTA	D	270
	IY=YI/DELTA	D	280
	IF (XI.LT.XMAX1) GO TO 101	D	290
	NUM(I)=(IY+1-IY/NINTY)*L4-1	D	300
	GO TO 103	D	310
101	IF (YI.LT.YMAX1) GO TO 102	D	320
	NUM(I)=L4*LY+4*IX+3	D	330
	GO TO 103	D	340
102	XN=XI-IX*DELTA	D	350
	YN=YI-IY*DELTA	D	360
	I1=1	D	370
	IF (XN.GT.YN) I1=2	D	380
	I2=0	D	390
	IF (YN.GT.(DELTA-XN)) I2=-5	D	400
	NUM(I)=IY*L4+4*IX+IABS(I1+I2)	D	410
103	CONTINUE	D	420
104	CONTINUE	D	430
	RETURN	D	440
C		D	450
C	END	D	460
C	SUBROUTINE SORT	D	470
C		D	480
C	-----	D	490
C	SORTING THE POINTS IN INCREASING ORDER	E	10
C	-----	E	20
C		E	30
C		E	40
C		E	50
C	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	E	60
	1,NP(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	E	70
	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	E	80
C		E	90
	JM=1	E	100
	N=NPGIV-1	E	110
	DO 105 I=1,N	E	120
	IA=I+1	E	130
	LI=I	E	140
	IF (NUM(IA).GE.NUM(LI)) GO TO 103	E	150
101	CONTINUE	E	160
	IF (NG(LI).EQ.0) GO TO 102	E	170
		E	180

	L1=LI	E	190
	LI=NG(LI)	E	200
	IF (NUM(LI).LE.NUM(LI)) GO TO 101	E	210
	NG(LI)=NG(LI)	E	220
	ND(LI)=ND(LI)	E	230
	NG(LI)=IA	E	240
	ND(LI)=IA	E	250
	GO TO 105	E	260
102	JM=IA	E	270
	NG(LI)=JM	E	280
	ND(LI)=LI	E	290
	GO TO 105	E	300
103	IF (ND(LI).EQ.0) GO TO 104	E	310
	LI=LI	E	320
	LI=ND(LI)	E	330
	IF ((NUM(LI)).GE.(NUM(LI))) GO TO 103	E	340
	ND(LI)=ND(LI)	E	350
	NG(LI)=NG(LI)	E	360
	ND(LI)=IA	E	370
	NG(LI)=IA	E	380
	GO TO 105	E	390
104	ND(LI)=IA	E	400
	NG(LI)=LI	E	410
105	CONTINUE	E	420
	DO 106 K=1,NPGIV	E	430
	NUMTRI(K)=NUM(JM)	E	440
	NUMTRI(K,2)=JM	E	450
	JM=ND(JM)	E	460
106	CONTINUE	E	470
	RETURN	E	480
C	END	E	490
	SUBROUTINE INITL	E	500
C		F	10
C		F	20
C		F	30
C	EVALUATION OF THE PLACE OCCUPIED BY THE FIRST NODE	F	40
C	OF A LINE INSIDE THE VECTOR NUMTRI	F	50
C		F	60
C		F	70
	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	F	80
	1,NP(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	F	90
	2MAX,NINTX,NINTY,HNDX,HNDY,L4,NENT(4,2)	F	100
	COMMON /SEC/ NUMEL(300),NTR(4,2),NHEIG(32),DIST(32),NHEIG(3),DIST	F	110
	1(3),LNK,RNY,KS,NBEO	F	120
C		F	130
	IND=1	F	140
	IND1=1	F	150
	L1=L4	F	160
	DO 104 I=1,NINTY	F	170
	IF (NUMTRI(IND).GT.L1) GO TO 103	F	180
	NP(IND)=IND	F	190
	DO 102 J=1,1000	F	200
	IND=IND+1	F	210
	IF (IND.LE.NPGIV) GO TO 101	F	220
	K=I+1	F	230
	GO TO 105	F	240
101	CONTINUE	F	250
	IF (NUMTRI(IND).GT.L1) GO TO 103	F	260
102	CONTINUE	F	270
103	L1=L1+L4	F	280
104	CONTINUE	F	290
	GO TO 107	F	300
105	CONTINUE	F	310
	DO 106 I=K,NINTY	F	320
	NP(IND)=0	F	330
106	CONTINUE	F	340
107	NP(IND)=99999	F	350
	RETURN	F	360
C		F	370
	END	F	380
	SUBROUTINE SEARCH (XT,YT,NX,NY)	G	10

C		G	20
C		G	30
C	SEARCH OF THE EIGHT GIVEN POINTS AROUND THE EXAMINED NODE	G	40
C		G	50
C		G	60
C	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),NB(300)	G	70
C	1,MPCINTE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	G	80
C	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	G	90
C	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG1(3),DIST	G	100
C	11(8),LNK,KNY,KS,NSEG	G	110
C	COMMON /B3/ NKRIQ,NADR	G	120
C		G	130
C	LNK=NINTX-NX	G	140
C	KNY=NINTY-NY	G	150
C	DO 101 I=1,4	G	160
C	DO 101 J=1,8	G	170
C	NTR(I,J)=0	G	180
C	101 CONTINUE	G	190
C	IF ((NX.GE.NNDX).OR.(NY.GE.NNDY)) GO TO 102	G	200
C		G	210
C		G	220
C	CALL SECT1 (NX,NY)	G	230
C		G	240
C		G	250
C		G	260
C	CALL SECT2 (NX,NY)	G	270
C		G	280
C		G	290
C		G	300
C	102 CONTINUE	G	310
C	IF ((NX.LE.0).OR.(NY.GE.NNDY)) GO TO 103	G	320
C		G	330
C		G	340
C		G	350
C	CALL SECT3 (NX,NY)	G	360
C		G	370
C		G	380
C		G	390
C	CALL SECT4 (NX,NY)	G	400
C		G	410
C		G	420
C		G	430
C	103 CONTINUE	G	440
C	IF ((NX.LE.0).OR.(NY.LE.0)) GO TO 104	G	450
C		G	460
C		G	470
C		G	480
C	CALL SECT5 (NX,NY)	G	490
C		G	500
C		G	510
C		G	520
C	CALL SECT6 (NX,NY)	G	530
C		G	540
C		G	550
C		G	560
C	104 CONTINUE	G	570
C	IF ((NY.LE.0).OR.(NX.GE.NNDX)) GO TO 105	G	580
C		G	590
C		G	600
C		G	610
C	CALL SECT7 (NX,NY)	G	620
C		G	630
C		G	640
C		G	650
C	CALL SECT8 (NX,NY)	G	660
C		G	670
C		G	680
C		G	690
C	105 CONTINUE	G	700
C	NKRIQ=0	G	710
C		G	720

C	NADR=0	G	730
C	-----	G	740
C	CALL EIGT (XT,YT,0)	G	750
C	-----	G	760
C	RETURN	G	770
C	END	G	780
C	SUBROUTINE SECT1 (NX,NY)	G	790
C	-----	G	800
C	SECTOR FROM 0. TO PI /4	G	810
C	-----	G	820
C	COMMON X(300),Y(300),Z(300),NUM(300),NUNTRI(300,2),NG(300),ND(300)	G	830
C	1,NPDIITE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	H	10
C	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	H	20
C	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG1(8),DIST	H	30
C	11(8),LNK,KNY,K8,NBEQ	H	40
C	-----	H	50
C	INUM=2	H	60
C	NTRDUV=0	H	70
C	NLIGN=NTRDUV	H	80
C	NCOL=NLIGN	H	90
C	NUMEL(1)=L4*NY+4*NX+2	H	100
C	NELEM=NUMEL(1)	H	110
C	101 NUMEL(INUM)=NUMEL(INUM-1)+1	H	120
C	-----	H	130
C	CALL LOOK1 (INUM,1,NLIGN,NY,NTRDUV,1)	H	140
C	-----	H	150
C	IF (NTRDUV.EQ.4) RETURN	H	160
C	102 CONTINUE	H	170
C	NCOL=NCOL+1	H	180
C	IF (NCOL.GE.LNX) RETURN	H	190
C	NLIGN=0	H	200
C	NC4=4*NCOL	H	210
C	NELEM4=NELEM+NC4-L4-1	H	220
C	DO 103 I=1,NCOL	H	230
C	INUM=4*I	H	240
C	NUMEL(INUM-3)=NELEM4+L4*I	H	250
C	NUMEL(INUM-2)=NUMEL(INUM-3)+1	H	260
C	NUMEL(INUM-1)=NUMEL(INUM-2)+1	H	270
C	NUMEL(INUM)=NUMEL(INUM-1)+1	H	280
C	NLIGN=NLIGN+1	H	290
C	IF (NLIGN.GE.KNY) GO TO 104	H	300
C	103 CONTINUE	H	310
C	INUM=INUM+2	H	320
C	NUMEL(INUM-1)=NUMEL(INUM-4)+L4	H	330
C	GO TO 101	H	340
C	-----	H	350
C	104 CALL LOOK1 (INUM,1,NLIGN,NY,NTRDUV,1)	H	360
C	-----	H	370
C	IF (NTRDUV.EQ.4) RETURN	H	380
C	IF (NCOL.LT.LNX) GO TO 102	H	390
C	RETURN	H	400
C	END	H	410
C	SUBROUTINE SECT4 (NX,NY)	H	420
C	-----	H	430
C		H	440
C		H	450
C		H	460
C		H	470
C		H	480
C		H	490
C		H	500
C		H	510
C		H	520
C		H	530
C		H	540
C		H	550
C		H	560
C		H	570
C		I	10
C		I	20
C		I	30

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C      SECTOR FROM 3 PI/4 TO PI
C      -----
C      COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)
1      NPDIINTE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y
2      MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)
C      COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG(8),DIST
11(8),LNK,KNY,K8,NBEQ
C
C      INUM=2
C      NTRDUV=0
C      NLIGN=NTRDUV
C      NCOL=NLIGN
C      NUMEL(1)=L4*NY+4*NX-3
C      NELEM=NUMEL(1)
101  NUMEL(INUM)=NUMEL(INUM-1)+1
C
C      -----
C      CALL LOOK1 (INUM,4,NLIGN,NY,NTRDUV,1)
C
C      -----
C      IF (NTRDUV.EQ.4) RETURN
102  CONTINUE
C      NCOL=NCOL+1
C      IF (NCOL.GE.NX) RETURN
C      NLIGN=0
C      NC4=4*NCOL
C      NELEM4=NELEM-L4-NC4
C      DO 103 I=1,NCOL
C        INUM=4*I
C        NUMEL(INUM-3)=NELEM4+L4*I
C        NUMEL(INUM-2)=NUMEL(INUM-3)+1
C        NUMEL(INUM-1)=NUMEL(INUM-2)+1
C        NUMEL(INUM)=NUMEL(INUM-1)+1
C        NLIGN=NLIGN+1
C        IF (NLIGN.GE.KNY) GO TO 104
103  CONTINUE
C      INUM=INUM+2
C      NUMEL(INUM-1)=NUMEL(INUM-4)+L4-1
C      GO TO 101
C
C      -----
104  CALL LOOK1 (INUM,4,NLIGN,NY,NTRDUV,1)
C
C      -----
C      IF (NTRDUV.EQ.4) RETURN
C      IF (NCOL.LT.NX) GO TO 102
C      RETURN
C
C      END
C      SUBROUTINE SECT5 (NX,NY)
C
C      -----
C      SECTOR FROM PI TO 5 PI/4
C      -----
C      COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)
1      NPDIINTE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y
2      MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)
C      COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG(8),DIST
11(8),LNK,KNY,K8,NBEQ
C
C      INUM=2
C      NTRDUV=0
C      NLIGN=NTRDUV
C      NCOL=NLIGN
C      NUMEL(1)=L4*NY-L4+4*NX

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I 40
I 50
I 60
I 70
I 80
I 90
I 100
I 110
I 120
I 130
I 140
I 150
I 160
I 170
I 180
I 190
I 200
I 210
I 220
I 230
I 240
I 250
I 260
I 270
I 280
I 290
I 300
I 310
I 320
I 330
I 340
I 350
I 360
I 370
I 380
I 390
I 400
I 410
I 420
I 430
I 440
I 450
I 460
I 470
I 480
I 490
I 500
I 510
I 520
I 530
I 540
I 550
I 560
I 570
J 10
J 20
J 30
J 40
J 50
J 60
J 70
J 80
J 90
J 100
J 110
J 120
J 130
J 140
J 150
J 160
J 170

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	NELEM=NUMEL(1)	J	180
101	NUMEL(INUM)=NUMEL(INUM-1)-3	J	190
C		J	200
C	-----	J	210
C	CALL LOOK1 (INUM,5,NLIGN,NY,NTRDUV,-1)	J	220
C		J	230
C	-----	J	240
C		J	250
	IF (NTRDUV.EQ.4) RETURN	J	260
102	CONTINUE	J	270
	NCOL=NCOL+1	J	280
	IF (NCOL.GE.NX) RETURN	J	290
	NLIGN=0	J	300
	NC4=4*NCOL	J	310
	NELEM4=NELEM-NC4+L4	J	320
	DO 103 I=1,NCOL	J	330
	INUM=4*I	J	340
	NUMEL(INUM-3)=NELEM4-L4*I	J	350
	NUMEL(INUM-2)=NUMEL(INUM-3)-1	J	360
	NUMEL(INUM-1)=NUMEL(INUM-2)-1	J	370
	NUMEL(INUM)=NUMEL(INUM-1)-1	J	380
	NLIGN=NLIGN+1	J	390
	IF (NLIGN.GE.NY) GO TO 104	J	400
103	CONTINUE	J	410
	INUM=INUM+2	J	420
	NUMEL(INUM-1)=NUMEL(INUM-5)-L4	J	430
	GO TO 101	J	440
C		J	450
C	-----	J	460
C		J	470
104	CALL LOOK1 (INUM,5,NLIGN,NY,NTRDUV,-1)	J	480
C		J	490
C	-----	J	500
C		J	510
	IF (NTRDUV.EQ.4) RETURN	J	520
	IF (NCOL.LT.NX) GO TO 102	J	530
	RETURN	J	540
C		J	550
	END	J	560
	SUBROUTINE SECT8 (NX,NY)	J	570
C		K	10
C	-----	K	20
C		K	30
C	SECTOR FROM 7 PI/4 TO 2 PI	K	40
C	-----	K	50
C		K	60
	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	K	70
	1,MPOINTE(300),NPLA(2),XENT(4),YENT(4),DELTA,MFGIV,XMIN,XMAX,YMIN,Y	K	80
	EMAX,MINTX,MINTY,MINDX,MINDY,L4,MENT(4,2)	K	90
	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG1(8),DIST	K	100
	11(8),LNX,KNY,K8,NBEQ	K	110
C		K	120
	INUM=2	K	130
	NTRDUV=0	K	140
	NLIGN=NTRDUV	K	150
	NCOL=NLIGN	K	160
	NUMEL(1)=L4*NY-L4+4*NX+4	K	170
	NELEM=NUMEL(1)	K	180
101	NUMEL(INUM)=NUMEL(INUM-1)-1	K	190
C		K	200
C	-----	K	210
C		K	220
	CALL LOOK1 (INUM,8,NLIGN,NY,NTRDUV,-1)	K	230
C		K	240
C	-----	K	250
C		K	260
	IF (NTRDUV.EQ.4) RETURN	K	270
102	CONTINUE	K	280
	NCOL=NCOL+1	K	290
	IF (NCOL.GE.LNX) RETURN	K	300
	NLIGN=0	K	310

	NC4=4*NCOL	K	320
	NELEM4=NC4+NELEM+L4	K	330
	DO 103 I=1,NCOL	K	340
	INUM=4*I	K	350
	NUMEL(INUM-3)=NELEM4-L4*I	K	360
	NUMEL(INUM-2)=NUMEL(INUM-3)-1	K	370
	NUMEL(INUM-1)=NUMEL(INUM-2)-1	K	380
	NUMEL(INUM)=NUMEL(INUM-1)-1	K	390
	NLIGN=NLIGN+1	K	400
	IF (NLIGN.GE.NY) GO TO 104	K	410
103	CONTINUE	K	420
	INUM=INUM+2	K	430
	NUMEL(INUM-1)=NUMEL(INUM-4)-L4+1	K	440
	GO TO 101	K	450
C		K	460
C		K	470
C		K	480
C	104 CALL LOOK1 (INUM,8,NLIGN,NY,NTRDUV,-1)	K	490
C		K	500
C		K	510
C		K	520
C	IF (NTRDUV.EQ.4) RETURN	K	530
C	IF (NCOL.LT.LNX) GO TO 102	K	540
C	RETURN	K	550
C		K	560
C	END	K	570
C	SUBROUTINE SECT2 (NX,NY)	L	10
C		L	20
C		L	30
C	SECTOR FROM PI/4 TO PI/2	L	40
C		L	50
C		L	60
C	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	L	70
C	1,NPDIINTE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	L	80
C	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	L	90
C	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIGI(8),DIST	L	100
C	11(8),LNX,KNY,K8,NBEQ	L	110
C		L	120
C	INUM=2	L	130
C	NTRDUV=0	L	140
C	NCOL=NTRDUV	L	150
C	NLIGN=NCOL	L	160
C	NUMEL(1)=L4*NY+4*NX+1	L	170
C	NELEM=NUMEL(1)	L	180
101	NUMEL(INUM)=NUMEL(INUM-1)+3	L	190
C		L	200
C		L	210
C		L	220
C	CALL LOOK2 (INUM,2,NLIGN,NY,NTRDUV,1)	L	230
C		L	240
C		L	250
C		L	260
C	IF (NTRDUV.EQ.4) RETURN	L	270
102	CONTINUE	L	280
	NLIGN=NLIGN+1	L	290
	IF (NLIGN.GE.KNY) RETURN	L	300
	NCOL=1	L	310
	NELEM4=NELEM+L4*NLIGN-4	L	320
	DO 103 I=1,NLIGN	L	330
	INUM=4*I	L	340
	NUMEL(INUM-3)=NELEM4+4*I	L	350
	NUMEL(INUM-2)=NUMEL(INUM-3)+1	L	360
	NUMEL(INUM-1)=NUMEL(INUM-2)+1	L	370
	NUMEL(INUM)=NUMEL(INUM-1)+1	L	380
	NCOL=NCOL+1	L	390
	IF (NCOL.GT.LNX) GO TO 104	L	400
103	CONTINUE	L	410
	INUM=INUM+2	L	420
	NUMEL(INUM-1)=NUMEL(INUM-2)+1	L	430
	GO TO 101	L	440
C		L	450

C	-----	L	460
C		L	470
C	104 CALL LOOK2 (INUM,2,NLIGN,NY,NTRDUV,1)	L	480
C		L	490
C	-----	L	500
C		L	510
C	IF (NTRDUV.EQ.4) RETURN	L	520
C	IF (NLIGN.LT.KNY) GO TO 102	L	530
C	RETURN	L	540
C		L	550
C	END	L	560
C	SUBROUTINE SECT3 (NX,NY)	M	10
C		M	20
C	-----	M	30
C	SECTOR FROM PI/2 TO 3 PI/4	M	40
C	-----	M	50
C		M	60
C	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	M	70
C	1,NPDIATE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	M	80
C	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	M	90
C	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG1(8),DIST	M	100
C	11(8),LNK,KNY,K8,NBEQ	M	110
C		M	120
C	INUM=2	M	130
C	NTRDUV=0	M	140
C	NCOL=NTRDUV	M	150
C	NLIGN=NCOL	M	160
C	NUMEL(1)=L4*NY+4*NX-1	M	170
C	NELEM=NUMEL(1)	M	180
C	101 NUMEL(INUM)=NUMEL(INUM-1)+1	M	190
C		M	200
C	-----	M	210
C		M	220
C	CALL LOOK2 (INUM,3,NLIGN,NY,NTRDUV,1)	M	230
C		M	240
C	-----	M	250
C		M	260
C	IF (NTRDUV.EQ.4) RETURN	M	270
C	102 CONTINUE	M	280
C	NLIGN=NLIGN+1	M	290
C	IF (NLIGN.GE.KNY) RETURN	M	300
C	NCOL=1	M	310
C	NLI4=L4*NLIGN	M	320
C	NELEM4=NLI4+NELEM+4	M	330
C	DO 103 I=1,NLIGN	M	340
C	INUM=4*I	M	350
C	NUMEL(INUM-3)=NELEM4-4*I	M	360
C	NUMEL(INUM-2)=NUMEL(INUM-3)-1	M	370
C	NUMEL(INUM-1)=NUMEL(INUM-2)-1	M	380
C	NUMEL(INUM)=NUMEL(INUM-1)+3	M	390
C	NCOL=NCOL+1	M	400
C	IF (NCOL.GT.NX) GO TO 104	M	410
C	103 CONTINUE	M	420
C	INUM=INUM+2	M	430
C	NUMEL(INUM-1)=NUMEL(INUM-4)-3	M	440
C	GO TO 101	M	450
C		M	460
C	-----	M	470
C		M	480
C	104 CALL LOOK2 (INUM,3,NLIGN,NY,NTRDUV,1)	M	490
C		M	500
C	-----	M	510
C		M	520
C	IF (NTRDUV.EQ.4) RETURN	M	530
C	IF (NLIGN.LT.KNY) GO TO 102	M	540
C	RETURN	M	550
C		M	560
C	END	M	570
C	SUBROUTINE SECT6 (NX,NY)	N	10
C		N	20
C	-----	N	30

C	SECTOR FROM 5 PI/2 TO 3 PI/2	N	40
C	-----	N	50
C	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	N	60
	1,NPOINTE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	N	70
	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	N	80
	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG1(8),DIST	N	90
	11(8),LNK,KNY,K8,NBEQ	N	100
C		N	110
	INUM=2	N	120
	NTRDUV=0	N	130
	NCOL=NTRDUV	N	140
	NLIGN=NCOL	N	150
	NUMEL(1)=L4*NY-L4+4*NX-1	N	160
	NELEM=NUMEL(1)	N	170
101	NUMEL(INUM)=NUMEL(INUM-1)-1	N	180
C		N	190
C	-----	N	200
C		N	210
C	CALL LOOK2 (INUM,6,NLIGN,NY,NTRDUV,-1)	N	220
C		N	230
C	-----	N	240
C		N	250
	IF (NTRDUV.EQ.4) RETURN	N	260
102	CONTINUE	N	270
	NLIGN=NLIGN+1	N	280
	IF (NLIGN.GE.NY) RETURN	N	290
	NCOL=1	N	300
	NLI4=NLIGN*L4	N	310
	NELEM4=NELEM-NLI4+4	N	320
	DO 103 I=1,NLIGN	N	330
	INUM=4*I	N	340
	NUMEL(INUM-3)=NELEM4-4*I	N	350
	NUMEL(INUM-2)=NUMEL(INUM-3)-1	N	360
	NUMEL(INUM-1)=NUMEL(INUM-2)-1	N	370
	NUMEL(INUM)=NUMEL(INUM-1)+3	N	380
	NCOL=NCOL+1	N	390
	IF (NCOL.GT.NX) GO TO 104	N	400
103	CONTINUE	N	410
	INUM=INUM+2	N	420
	NUMEL(INUM-1)=NUMEL(INUM-4)-3	N	430
	GO TO 101	N	440
C		N	450
C	-----	N	460
C		N	470
C	104 CALL LOOK2 (INUM,6,NLIGN,NY,NTRDUV,-1)	N	480
C		N	490
C	-----	N	500
C		N	510
	IF (NTRDUV.EQ.4) RETURN	N	520
	IF (NLIGN.LT.NY) GO TO 102	N	530
	RETURN	N	540
C		N	550
	END	N	560
	SUBROUTINE SECT7 (NX,NY)	N	570
C		O	10
C	-----	O	20
C		O	30
C	SECTOR FROM 3 PI/2 TO 7 PI/4	O	40
C	-----	O	50
C		O	60
	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	O	70
	1,NPOINTE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	O	80
	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	O	90
	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG1(8),DIST	O	100
	11(8),LNK,KNY,K8,NBEQ	O	110
C		O	120
	INUM=2	O	130
	NTRDUV=0	O	140
	NCOL=NTRDUV	O	150
	NLIGN=NCOL	O	160
	NUMEL(1)=L4*NY-L4+4*NX+1	O	170

	NELEM=NUMEL(1)	D	180
101	NUMEL(INUM)=NUMEL(INUM-1)+1	D	190
C		D	200
C	-----	D	210
C	CALL LOOK2 (INUM,7,NLIGN,NY,NTRDUV,-1)	D	220
C		D	230
C	-----	D	240
C		D	250
	IF (NTRDUV.EQ.4) RETURN	D	260
102	CONTINUE	D	270
	NLIGN=NLIGN+1	D	280
	IF (NLIGN.GE.NY) RETURN	D	290
	NCOL=1	D	300
	NLI4=L4*NLIGN	D	310
	NELEM4=NELEM-NLI4-4	D	320
	DO 103 I=1,NLIGN	D	330
	INUM=4*I	D	340
	NUMEL(INUM-3)=NELEM4+4*I	D	350
	NUMEL(INUM-2)=NUMEL(INUM-3)+1	D	360
	NUMEL(INUM-1)=NUMEL(INUM-2)+1	D	370
	NUMEL(INUM)=NUMEL(INUM-1)+1	D	380
	NCOL=NCOL+1	D	390
	IF (NCOL.GT.LNX) GO TO 104	D	400
103	CONTINUE	D	410
	INUM=INUM+2	D	420
	NUMEL(INUM-1)=NUMEL(INUM-2)+1	D	430
	GO TO 101	D	440
C		D	450
C	-----	D	460
C		D	470
104	CALL LOOK2 (INUM,7,NLIGN,NY,NTRDUV,-1)	D	480
C		D	490
C	-----	D	500
C		D	510
	IF (NTRDUV.EQ.4) RETURN	D	520
	IF (NLIGN.LT.NY) GO TO 102	D	530
	RETURN	D	540
C		D	550
	END	D	560
	SUBROUTINE LOOK1 (INUM,NUMSEC,NLIGN,NY,NTRDUV,KDIR)	P	570
C		P	10
C		P	20
C	-----	P	30
C	SEARCHING THE GIVEN INFORMATION IN SECTORS 1,4,5,8	P	40
C	-----	P	50
C		P	60
	COMMON X(300),Y(300),Z(300),NUM(300),NUMTR(300,2),NG(300),ND(300)	P	70
	1,NP(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	P	80
	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	P	90
	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG1(8),DIST	P	100
	11(8),LNX,KNY,KS,NBEQ	P	110
C		P	120
	KR=1	P	130
	NOLI=NY+(1-KDIR)/2	P	140
	IF (NLIGN.EQ.0) GO TO 107	P	150
	DO 106 I=1,NLIGN	P	160
	NOLI=NOLI+KDIR	P	170
	IF (NOLI.GE.NNDY) RETURN	P	180
	NUMP=NP(300,NOLI)	P	190
	IF (NUMP.NE.0) GO TO 101	P	200
	KR=KR+4	P	210
	GO TO 106	P	220
101	NUMP4=NUMP+L4	P	230
	NUMPA=MIN0(NUMP4,NPGIV)	P	240
	JJ=0	P	250
	DO 105 J=1,100	P	260
	JJ=JJ+1	P	270
	IF (JJ.GE.5) GO TO 106	P	280
	DO 103 J1=NUMP,NUMPA	P	290
	IF (NUMTR(J1,1)-NUMEL(KR)) 103,102,104	P	300
102	NTRDUV=NTRDUV+1	P	310

	NTR(NTRDUV,NUMSEC)=NUMTRI(J,2)	P	320
	KR=KR-1	P	330
	JJ=JJ-1	P	340
	NUMP=J+1	P	350
	IF (NTRDUV.EQ.4) RETURN	P	360
	GO TO 105	P	370
103	CONTINUE	P	380
104	NUMP=NPOINTE(NOLI)	P	390
105	KR=KR+1	P	400
106	CONTINUE	P	410
	IF (KR.GT.INUM) RETURN	P	420
107	NOLI=NY+KDIR*NLIGN+(1+KDIR)/2	P	430
	IF (NOLI.GE.NNDY) RETURN	P	440
	NUMP=NPOINTE(NOLI)	P	450
	IF (NUMP.EQ.0) RETURN	P	460
	NUMP4=NUMP+L4	P	470
	NUMPA=MIN0(NPGIV,NUMP4)	P	480
	II=0	P	490
	DO 111 I=1,100	P	500
	II=II+1	P	510
	IF (II.GE.3) GO TO 112	P	520
	DO 109 J=NUMP,NUMPA	P	530
	IF (NUMTRI(J,1)-NUMEL(KR)) 109,108,110	P	540
108	NTRDUV=NTRDUV+1	P	550
	KR=KR-1	P	560
	II=II-1	P	570
	NUMP=J+1	P	580
	NTR(NTRDUV,NUMSEC)=NUMTRI(J,2)	P	590
	IF (NTRDUV.EQ.4) RETURN	P	600
	GO TO 111	P	610
109	CONTINUE	P	620
110	NUMP=NPOINTE(NOLI)	P	630
111	KR=KR+1	P	640
112	CONTINUE	P	650
	RETURN	P	660
C	END	P	670
	SUBROUTINE LOOK2 (INUM,NUMSEC,NLIGN,NY,NTRDUV,KDIR)	P	680
C		Q	10
C		Q	20
C	SEARCHING THE GIVEN INFORMATION IN SECTORS 2,3,6,7	Q	30
C		Q	40
C		Q	50
C		Q	60
	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	Q	70
	1,NPOINTE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	Q	80
	2MAX,NINTX,NINTY,NNDX,NNDY,L4,MENT(4,2)	Q	90
	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG1(8),DIST	Q	100
	11(8),LNK,KNY,K8,NBEQ	Q	110
C		Q	120
	NOLI=NY+KDIR*NLIGN+(1+KDIR)/2	Q	130
	IF (NOLI.GE.NNDY) RETURN	Q	140
	NUMP=NPOINTE(NOLI)	Q	150
	IF (NUMP.EQ.0) RETURN	Q	160
	NUMP4=NUMP+L4	Q	170
	NUMPA=MIN0(NUMP4,NPGIV)	Q	180
	IN=0	Q	190
	DO 104 I=1,160	Q	200
	IN=IN+1	Q	210
	IF (IN.GT.INUM) RETURN	Q	220
	DO 102 J=NUMP,NUMPA	Q	230
	IF (NUMTRI(J,1)-NUMEL(IN)) 102,101,103	Q	240
101	NTRDUV=NTRDUV+1	Q	250
	NTR(NTRDUV,NUMSEC)=NUMTRI(J,2)	Q	260
	IF (NTRDUV.EQ.4) RETURN	Q	270
	IN=IN-1	Q	280
	NUMP=J+1	Q	290
	GO TO 104	Q	300
102	CONTINUE	Q	310
103	NUMP=NPOINTE(NOLI)	Q	320
104	CONTINUE	Q	330
	RETURN	Q	340

C	END	Q	350
	SUBROUTINE EIGT (XT,YT,NFLAG)	Q	360
C		R	10
C		R	20
C	-----	R	30
C	DEFINES THE EIGHT CLOSEST INFORMATIONS TO THE EXAM. NODE	R	40
C	-----	R	50
	COMMON X(300),Y(300),Z(300),NUM(300),NUMTRI(300,2),NG(300),ND(300)	R	60
	1,NPOINTE(300),NPLA(2),XENT(4),YENT(4),DELTA,NPGIV,XMIN,XMAX,YMIN,Y	R	70
	2MAX,NINTX,NINTY,NNDX,NNDY,L4,NENT(4,2)	R	80
	COMMON /SEC/ NUMEL(300),NTR(4,8),NHEIG(32),DIST(32),NHEIG1(8),DIST	R	90
	11(8),LNK,KNY,K8,NBEQ	R	100
	COMMON /B3/ NKRIG,NADR	R	110
	DIMENSION NHG(8)	R	120
C		R	130
C		R	140
	IF (NFLAG.NE.0) GO TO 110	R	150
	DO 101 I=1,32	R	160
	DIST(I)=0.	R	170
	NHEIG(I)=0	R	180
101	CONTINUE	R	190
	K8=0	R	200
	DO 102 I=1,8	R	210
	NTRI=NTR(1,I)	R	220
	IF (NTRI.EQ.0) GO TO 102	R	230
	K8=K8+1	R	240
	NHEIG(K8)=NTRI	R	250
102	CONTINUE	R	260
	DO 103 I=1,K8	R	270
	NSI=NHEIG(I)	R	280
	DIST(I)=SQRT((X(NSI)-XT)**2+(Y(NSI)-YT)**2)	R	290
103	CONTINUE	R	300
	DO 107 J=2,4	R	310
	JN=0	R	320
	JJ=0	R	330
	DO 104 I=1,8	R	340
	NTRI=NTR(J,I)	R	350
	IF (NTRI.EQ.0) GO TO 104	R	360
	JN=JN+1	R	370
	NHG(JN)=NTRI	R	380
C		R	390
	JJ=JJ+1	R	400
	NHEIG1(JJ)=NTRI	R	410
	DIST1(JJ)=SQRT((X(NTRI)-XT)**2+(Y(NTRI)-YT)**2)	R	420
104	CONTINUE	R	430
	JJ1=JJ-1	R	440
	DO 105 I=1,JJ1	R	450
	IP1=I+1	R	460
	DO 105 K=IP1,JJ	R	470
	IF (DIST1(K).GE.DIST1(I)) GO TO 105	R	480
	DIST1I=DIST1(I)	R	490
	NHEIG1I=NHEIG1(I)	R	500
	DIST1(I)=DIST1(K)	R	510
	NHEIG1(I)=NHEIG1(K)	R	520
	DIST1(K)=DIST1I	R	530
	NHEIG1(K)=NHEIG1I	R	540
105	CONTINUE	R	550
	DO 106 I=1,JJ	R	560
	K8=K8+1	R	570
	NHEIG(K8)=NHEIG1(I)	R	580
	DIST(K8)=DIST1(I)	R	590
106	CONTINUE	R	600
107	CONTINUE	R	610
C		R	620
	DO 108 I=1,6	R	630
	NHEIG(8+I)=NHG(I)	R	640
108	CONTINUE	R	650
	DO 109 I=9,14	R	660
	IF (NHEIG(I).GT.0) GO TO 109	R	670
	NHEIG(I)=NHEIG(I-8)	R	680
		R	690

109	CONTINUE	R	700
C		R	710
C		R	720
	RETURN	R	730
110	CONTINUE	R	740
	NH=NHEIG(NADR)	R	750
	NHEIG(NADR)=NHEIG(NGIV)	R	760
	NHEIG(NGIV)=NH	R	770
	IF (NHEIG(NADR).NE.0) RETURN	R	780
	NADR=NADR-1	R	790
	IF (NADR.NE.0) GO TO 111	R	800
	NSEQ=K3	R	810
	RETURN	R	820
111	NGIV=9	R	830
	GO TO 110	R	840
C		R	850
	END	R	860
	SUBROUTINE GAUSS (IC,N,C,B,V)	S	10
C		S	20
C		S	30
C	N = NUMBER OF EQUATIONS	S	40
C	C = MATRIX OF COEFFICIENTS	S	50
C	B = CONSTANT VECTOR	S	60
C	V = SOLUTION VECTOR	S	70
C		S	80
C		S	90
	DIMENSION C(IC,IC), B(IC), V(IC)	S	100
	DATA LEVEL/1/	S	110
C		S	120
C		S	130
	IF (LEVEL.EQ.1) GO TO 102	S	140
	DO 101 I=1,N	S	150
	PRINT 112, (C(I,J),J=1,N),B(I)	S	160
101	CONTINUE	S	170
102	CONTINUE	S	180
C		S	190
	LAST=N-1	S	200
	DO 103 I=1, LAST	S	210
	M=I	S	220
	ITEMP=I+1	S	230
	DO 103 J=ITEMP,N	S	240
C		S	250
C	FIND THE ELEMENT IN THE I-TH COLUMN WITH THE MAXIM.	S	260
C	ABSOLUTE VALUE FOR SCALING	S	270
C	=====	S	280
C		S	290
	IF (ABS(C(M,I)).GT.ABS(C(J,I))) GO TO 103	S	300
	M=J	S	310
103	CONTINUE	S	320
	IF (M.EQ.I) GO TO 105	S	330
C		S	340
C	CHANGE M-TH ROW TO I-TH ROW SO THAT MAX ELEMENT APPEARS	S	350
C	ON THE DIAGONAL	S	360
C	=====	S	370
C		S	380
	DO 104 J=I,N	S	390
	TEMP=C(M,J)	S	400
	C(M,J)=C(I,J)	S	410
104	C(I,J)=TEMP	S	420
C		S	430
C	CONSTANT VECTOR IS CHANGED	S	440
C		S	450
	TEMP=B(M)	S	460
	B(M)=B(I)	S	470
	B(I)=TEMP	S	480
105	DO 107 J=ITEMP,N	S	490
C		S	500
C	MULTIPLY AND SUBTRACT SO THAT ALL ELEMENTS BELOW	S	510
C	THE DIAGONAL TERM ARE ZERO	S	520
C	=====	S	530
C		S	540